Contrôlabilité des points d'explosion pour l'équation de la chaleur

Hatem ZAAG

CNRS et LAGA Université Sorbonne Paris Nord

GdT Contrôle
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En collaboration avec :
Ping Lin (Northeast Normal University, Changchun, Chine).

Contents

1 Introduction
- PDEs and blowup

2 The control question

3 Blowup point controllability

4 The construction problem with a localized source

Introduction: PDEs and blowup

• It is generally known that solutions to some types of partial differential equations have the behavior of blowup. Roughly speaking, blowup is a conception which means that a solution is unbounded in finite time.

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Introduction: PDEs and blowup

• It is generally known that solutions to some types of partial differential equations have the behavior of blowup. Roughly speaking, blowup is a conception which means that a solution is unbounded in finite time.

• In certain cases, the blowup of a solution is desired. For instance, the dramatic increase in temperature leads to the ignition of a chemical reaction.

• However, solutions to linear partial differential equations without control generally globally exist. It is naturally appealing to study if one can use feedback controls to these equations such that the corresponding solutions blow up in finite time and at given place.

Contents

1 Introduction

2 The control question
- The equation
- Formulation of the control problem
- Feedback control
- Controllability: the classical context and ours
- Controllability and blowup in parabolic equations: A bibliography

3 Blowup point controllability

4 The construction problem with a localized source

The equation

Let Ω be a bounded domain of $\mathbb{R}^n$ with smooth boundary $\partial\Omega$. This talk concerns a controllability problem for blowup points on the following heat equation.

$$\begin{cases} y_t - \Delta y = \mathbb{1}_\omega u, & x \in \Omega,\ t > 0, \\ y = 0, & x \in \partial\Omega,\ t > 0, \\ y(x,0) = y_0(x), & x \in \Omega. \end{cases} \tag{1}$$

Here the control u acts on a nonempty and open subset $\omega \subset \Omega$, and $\mathbb{1}_\omega$ is the characteristic function of the set ω.

Formulation of the control problem

- This is the question we consider:

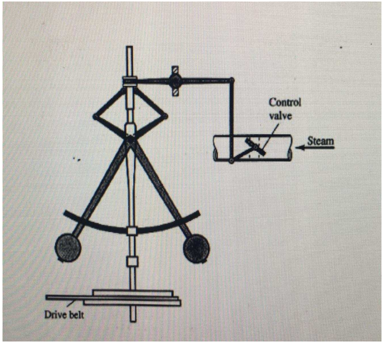
In the absence of control, the solution to the linear heat system globally exists in a bounded domain. While, for a given time $T > 0$ and a point a in this domain, we find a feedback control $u = F(y)$, which is acted on an internal subset of this domain, such that the corresponding solution to system (1) blows up at time T and has a unique blowup point a.

Feedback control

- From the point of view of applied science, feedback control can form a closed-loop system and plays an effective role of control.

A historical example

- In 1788, Watt invented the steam speed regulator based on the principle of feedback control. The efficiency and performance of the steam engine are greatly improved. It became the symbol of the first industrial revolution.



Watt Governor (1788)

Controllability: the classical context and ours

• In the past decades, classical controllability problems have been studied in the literature. Generally, a system of differential equations is called controllable, if one can act a control to this system such that the corresponding solution reaches or sufficiently approximates to a certain target at a given time.

Controllability: the classical context and ours

• In the past decades, classical controllability problems have been studied in the literature. Generally, a system of differential equations is called controllable, if one can act a control to this system such that the corresponding solution reaches or sufficiently approximates to a certain target at a given time.

• Our problem differs with classical controllability ones.

The target of our controllability problem is “infinity”, which is outside the state space of the solutions. The purpose to act a control on the system considered in this paper is to make the corresponding solution blow up at given time, while the targets of classical controllability problems are within the state space.

Controllability and blowup in parabolic equations: A bibliography

In recent years, the controllability of equations with the property of blowup attracted many people's interest. References [1, 2] concern the controllability of weakly blowing up semilinear parabolic equations.

[1] A. Boubova, E. Fernández-Cara, M. González-Burgos, E. Zuazua, On the controllability of parabolic systems with a nonlinear term involving the state and the gradient. SIAM J. Control Optim., 41(3) (2003)., 798-819.

[2] E. Fernández-Cara, E. Zuazua, Null and approximate controllability for weakly blowing up semilinear heat equations. Ann. Inst. H. Poincaré Anal. NonLinéaire, 17 (2000), 583-616.

A result by Fernández-Cara and Zuazua

For instantce, the following controlled system in [2] was considered,

$$\begin{cases} y_t - \Delta y + f(y, \nabla y) = \mathbb{1}_\omega u, & x \in \Omega \times (0,T), \\ y = 0, & x \in \partial\Omega \times (0,T), \\ y(x,0) = y_0(x), & x \in \Omega. \end{cases} \tag{2}$$

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• If f is locally Lipschitz-continuous, $f(0,0) = 0$ and

$$\lim_{|(s,p)|\to\infty} \frac{|g(s,p)|}{\log^{3/2}(1+|s|+|p|)} = 0, \quad \lim_{|(s,p)|\to\infty} \frac{|G(s,p)|}{\log^{1/2}(1+|s|+|p|)} = 0,$$

where $g(s,p) = \int_0^1 \frac{\partial f}{\partial s}(\lambda s, \lambda p)d\lambda$, $G_i(s,p) = \int_0^1 \frac{\partial f}{\partial p_i}(\lambda s, \lambda p)d\lambda$. $1 \le i \le N$, then (2) is null controllable at any $T > 0$.

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• Blowup occurs in the absence of control, while the solution can be steered to be zero or to be sufficiently approximate to a given target in the state space at given time using controls.

Some recent papers

See also the recent papers:

[3] J.A. Bárcena-Petisco, Null controllability of the heat equation in pseudo-cylinders by an internal control, ESAIM Control Optim. Calc. Var. 26 (2020), 1-34.
[4] K. Kassab, Null controllability of semi-linear fourth order parabolic equations, J. Math. Pures Appl. 136(9) (2020), 279-312.
[5] K. Le Balc'h, Global null-controllability and nonnegative-controllability of slightly superlinear heat equations, J. Math. Pures Appl. 135(9) (2020), 103-139.

• The aim in these references is to prevent blowup by (open-loop) controls, which is different with the intention of making the solution blow up at given time in this paper.

Papers on feedback null controllability

• In the past decades, there are some references concerned with the problem of feedback null controllability, see, for instance,

[6] S. Chen, I. Lasiecka, Feedback exact null controllability for unbounded control problems in hilbert space, Journal of optimization theory and applications 74(2) (1992), 191¨C219.
[7] J.M. Coron, H.M. Nguyen, Null controllability and finite time stabilization for the heat equations with variable coefficients in space in one dimension via backstepping approach, Archive for Rational Mechanics and Analysis 225(3) (2017), 993¨C1023.
[8] M. Sîrbu, A Riccati equation approach to the null controllability of linear systems, Communications in Applied Analysis 6(2) (2002), 163-177.

Feedback blowup controllability

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Feedback blowup controllability

• As for feedback blowup controllability, Lin [9] considered the blowup controllability of the heat equation with feedback controls.

• It was proved in [9] that for any initial data in $H_0^1(\Omega)$ and for any time $T > 0$, there exist a number p with $1 < p < \infty$ and a feedback control acting on an internal subset of the space domain such that the L^{p+1} norm of the corresponding solution blows up at T.

[9] P. Lin, Global blowup controllability of heat equation with feedback control, Communications in Contemporary Mathematics 20(5) (2018), 1750062-1-11.

Feedback blowup controllability

• Han, Liu and Lin [10] derived a global exact blowup controllability for ordinary differential system $y'(t) = Ay(t) + Bu(t)$ in the case that (A, B) is null controllable, A and B are time-invariant matrix.

• More precisely, for any initial data in $\mathbb{R}^n$ and for any time $T > 0$, one can find a feedback control u to make the solution $y(\cdot)$ to this ODE system blow up at time $T > 0$, i.e., $\lim_{t\to T} |y(t)|_{\mathbb{R}^n} = +\infty$.

[10] S. Han, H. Liu and P. Lin, Null controllability and global blowup controllability of ordinary differential equations with feedback controls, J. Math. Anal. Appl. 493 (2021), 124510, 33 pp..

Contents

1 Introduction

2 The control question

3 Blowup point controllability
- Formulation of the problem
- Main results
- A noncontrollability result

4 The construction problem with a localized source

Blowup point controllability: formulation of the problem

Definition

Let $T > 0$ and $a \in \Omega$. We say that T is the blowup time and a is a blowup point of the solution y to system (1) with a feedback control u, if there are sequence $\{a_j\}_{j=1}^{\infty}$ with $a_j \to a$ and sequence $\{t_j\}_{j=1}^{\infty}$ with $t_j \to T$ such that $|y(a_j, t_j)| \to +\infty$, as $j \to \infty$.

This is our problem:

Problem (P) Given an initial data $y_0 \in H_0^1(\Omega)$, $a \in \Omega$ and $T > 0$, we find a feedback control u such that T is the blowup time of the corresponding solution y to (1), and y has unique blowup point a.

Let us recall the equation:

$$\begin{cases} y_t - \Delta y = \mathbb{1}_\omega u, & x \in \Omega,\ t > 0, \\ y = 0, & x \in \partial\Omega,\ t > 0, \\ y(x,0) = y_0(x), & x \in \Omega. \end{cases}$$

Here the control u acts on a nonempty and open subset $\omega \subset \Omega$, and $\mathbb{1}_\omega$ is the characteristic function of the set ω.

Controllability of the blowup point

Theorem 1.1 (Control Result) For any $y_0 \in H_0^1(\Omega)$, any $a \in \omega$ and any $T > 0$, there exist $T_1 \in (0, T/2)$ and $\widetilde{y}_0 \in C_0^\infty(\omega)$ such that the solution y to (1) with the following feedback control

$$u(x,t) := \begin{cases} -\mathbb{1}_\omega{}^* P(t)\big(y(t) - \widetilde{y}_0\big)(x) - \Delta \widetilde{y}_0(x), & (x,t) \in \Omega \times (0, T - T_1), \\ |y|^{p-1} y(x,t), & (x,t) \in \Omega \times [T - T_1, T), \end{cases}$$

exists on $[0, T)$, T is the blowup time of y and y has unique blowup point a, where $p > 1$. Here, $P \in C_S\big([0, T - T_1); \Sigma^+(L^2(\Omega))\big)$ is the unique mild solution to the following Riccati system,

$$\begin{cases} P'(t) + \Delta P(t) + P(t)\Delta - P(t)\mathbb{1}_\omega P(t) = 0 \text{ on } [0, T - T_1), \\ \lim\limits_{(s,z) \to (T - T_1, z_0)} \langle P(s) z, z \rangle = +\infty, \text{ for each } z_0 \in L^2(\Omega) \text{ and } z_0 \neq 0. \end{cases} \tag{3}$$

Rk. $\Sigma^+(L^2(\Omega))$ is the Banach space of all symmetric and positive operators acting in $L^2(\Omega)$. $C_S([0, T - T_1); \Sigma^+(H))$ is the set of all mappings $S : [0, T - T_1) \to \Sigma^+(H)$ such that $S(\cdot) z_0$ is continuous on $[0, T - T_1)$ for each $z_0 \in L^2(\Omega)$.

Blowup profile

Corollary (Blowup Profile) For all $R > 0$, as $t \to T$,

$$\sup_{\left\{|x-a|\le R\sqrt{(T-t)|\log(T-t)|}\right\}} \left|(T-t)^{\frac{1}{p-1}} y(x,t) - f\left(\frac{x-a}{\sqrt{(T-t)|\log(T-t)|}}\right)\right| \to 0,$$

where

$$f(\eta) = \left(p - 1 + \frac{(p-1)^2}{4p}|\eta|^2\right)^{-\frac{1}{p-1}}.$$

Controllability of the blowup point: Idea of the proof

It relies on 2 arguments:

- *A Nonlinear Construction Argument*, where we construct a blowup solution to some parabolic equation with a *localized* nonlinearity.

Controllability of the blowup point: Idea of the proof

It relies on 2 arguments:

- *A Nonlinear Construction Argument*, where we construct a blowup solution to some parabolic equation with a *localized* nonlinearity.
- *A Linear Control Argument*, where we connect the given initial state y_0 to the initial data of the Construction Step.

Step 1: A Nonlinear Construction Argument

In order to prove our main result Theorem 1.1 (Control Result), we first construct a blowup solution with prescribed profile for the following PDE:

$$\begin{cases} y_t - \Delta y = \mathbb{1}_\omega |y|^{p-1} y, & x \in \Omega, \ t > 0, \\ y = 0, & x \in \partial\Omega, \ t > 0, \\ y(x,0) = y_0(x), & x \in \Omega, \end{cases} \tag{4}$$

where $p > 1$ is arbitrary but fixed. This will play an important role in the proof of our main result. More precisely, we have the following theorem:

Step 1: A blowup solution with a localized nonlinearity

Theorem 1.2 (Construction Result) For any $a \in \omega$, there exists $T_0 > 0$ such that for any $T \in (0, T_0)$, there exists an initial data $y_0 \in C_0^\infty(\omega)$ such that the corresponding solution y to (4) exists on $[0, T)$, T is the blowup time of y and y has unique blowup point a. Moreover, for all $R > 0$,

$$\sup_{\left\{|x-a|\le R\sqrt{(T-t)|\log(T-t)|}\right\}} \left|(T-t)^{\frac{1}{p-1}} y(x,t) - f\left(\frac{x-a}{\sqrt{(T-t)|\log(T-t)|}}\right)\right| \to 0,$$

as $t \to T$, where

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Idea of the proof: Thanks to some cut-off function around $a \in \omega$, we recover the $\mathbb{R}^n$ case, however, with cut-off terms:

$$y_t - \Delta y = |y|^{p-1} y + \text{ cut-off terms, with } (x,t) \in \mathbb{R}^n \times [0, T)$$

Step 1: Applying the Construction Result to derive our main result

Step 1: Using the Construction Result (*A Nonlinear Argument*)

First, for any $T > 0$ and any $a \in \omega$, we will make use of Theorem 1.2 to find a special initial data supported in ω such that the corresponding solution blows up in time T_1 with $0 < T_1 < T/2$ at one blowup point a, with the prescribed blowup profile.

Step 2: A linear feedback null controllability result

Step 2: A Linear Feedback Null Controllability Result (*A Linear Argument*): Just as we did in Lin [9], we will prove that for each initial data, using the feedback null controllability results for linear heat equations obtained in [8], there exists a feedback control such that the corresponding solution can reach the above-mentioned special initial data constructed in the first step at $T - T_1$.

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Rk. According to [8], the null controllability was first conjectured by Fursikov in 1992. It was proved later by has been proved first by Lebeau and Robbiano in 1995, then by Fursikov and Imanuvilov in 1996. In [8], we have a solution of some "optimal control" problem.

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Conclusion: Combining these two steps, we can get our desired global blowup controllability for blowup points with feedback controls.

[8] M. Sîrbu, A Riccati equation approach to the null controllability of linear systems, Communications in Applied Analysis 6(2) (2002), 163-177.
[9] P. Lin, Global blowup controllability of heat equation with feedback control, Communications in Contemporary Mathematics 20(5) (2018), 1750062-1-11.

A noncontrollability result

We also have the following noncontrollability result:

Theorem 1.3 Suppose that $y_0 \in H_0^1(\Omega) \bigcap L^\infty(\Omega)$. If y is a corresponding solution to system (1) for some (open-loop or feedback) control and belongs to $C([0, t_{max}); L^\infty(\Omega))$, then any $a \in \Omega \setminus \overline{\omega}$ could not be the unique blowup point of y. Here, $[0, t_{max})$ denotes the maximal interval of existence of y.

Idea of Proof of The noncontrollability result

We prove Theorem 1.3 by contradiction. Suppose that $a \in \Omega \setminus \overline{\omega}$ is the unique blowup point of y. Then, since the control is localized in ω, one can find a ball neighborhood of a in $\Omega \setminus \overline{\omega}$ such that the control has no action in it. Hence, for any open-loop or feedback control, the corresponding solution to (1) is bounded in this ball. This contradicts the assumption that a is a blowup point.

Contents

1 Introduction

2 The control question

3 Blowup point controllability

4 The construction problem with a localized source
- The statement
- Idea of the proof

The result

Let us recall the PDE:

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where $p > 1$, $\omega \subset \Omega$.

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Let us recall the result:

Theorem 1.2 (Construction Result) For any $a \in \omega$, there exists $T_0 > 0$ such that for any $T \in (0, T_0)$, there exists an initial data $y_0 \in C_0^\infty(\omega)$ such that the corresponding solution y to (4) exists on $[0, T)$, T is the blowup time of y and y has unique blowup point a. Moreover, for all $R > 0$,

$$\sup_{\left\{|x-a| \le R\sqrt{(T-t)|\log(T-t)|}\right\}} \left| (T-t)^{\frac{1}{p-1}} y(x,t) - f\left(\frac{x-a}{\sqrt{(T-t)|\log(T-t)|}} \right) \right| \to 0,$$

as $t \to T$, where

$$f(\eta) = \left(p - 1 + \frac{(p-1)^2}{4p} |\eta|^2 \right)^{-\frac{1}{p-1}},\ \forall\, \eta \in \mathbb{R}.$$

Reduction to the whole space case

Consider any $a \in \omega$. Consider then $T > 0$ to be fixed small enough and $y \in C(\Omega \times [0, T))$ to be constructed such that y blows up at time $T > 0$ only at the point a.

Reduction to the whole space case

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Consider $\chi_0 \in C_0^\infty(\mathbb{R}_+)$ such that

$$\chi_0(\xi) = 1 \text{ if } 0 \leq \xi \leq 1, \quad \chi_0(\xi) = 0 \text{ if } \xi \geq 1.$$

Introduce

$$y_{int} = y(x,t)\chi_1(x), \quad y_{ext} = y(x,t)(1 - \chi_2(x)), \text{ where } \chi_i(x) = \chi_0\left(\frac{x-a}{\epsilon_i}\right),$$

$\epsilon_2 \leq \frac{1}{2}\epsilon_1$, $\epsilon_1 \leq \frac{1}{2}d(a, \partial\omega)$ (note the overlapping).

Equations on y_{int} and y_{ext}

Note that:

- y_{int} is supported in $B(a, \epsilon_1) \subset \omega$ and y_{ext} is supported in $\mathbb{R}^n \backslash B(a, \epsilon_2)$.

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Note that:

- y_{int} is supported in $B(a,\epsilon_1) \subset \omega$ and y_{ext} is supported in $\mathbb{R}^n \backslash B(a,\epsilon_2)$.
- For all $(x,t) \in \mathbb{R}^n \times [0,T)$,

$$\begin{aligned} \partial_t y_{int} &= \Delta y_{int} + |y_{int}|^{p-1} y_{int} + F_{int}, \\ \partial_t y_{ext} &= \Delta y_{ext} + F_{ext}, \end{aligned}$$

where

$$\begin{aligned} F_{int} &= 2\mathrm{div}(y \nabla \chi_1) - y \Delta \chi_1 + y_{int}(\mathbb{1}_\omega |y|^{p-1} - |z|^{p-1}), \\ F_{ext} &= -2\mathrm{div}(y \nabla \chi_2) + y \Delta \chi_2 + y_{ext} \mathbb{1}_\omega |y|^{p-1}. \end{aligned}$$

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- F_{int} and F_{ext} are supported outside $B(a,\epsilon_2)$.

Control of y_{int} and y_{ext}

Control of y_{int} and y_{ext}:

- y_{int} will be close to the blowup solution of $z_t = \Delta z + |z|^{p-1}z + C$, with $(T-t)^{\frac{1}{p-1}} z(x,t) \sim f\left(\frac{x-a}{\sqrt{(T-t)|\log(T-t)|}}\right)$;

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Condition: F_{int} and F_{ext} should be bounded.

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Condition: F_{int} and F_{ext} should be bounded.

This is possible, if $T > 0$ is small. Indeed, in that case, by construction, y_{int} will be

- unbounded *in* $B(a, \frac{\epsilon_2}{2})$,
- bounded *outside* $B(a, \frac{\epsilon_2}{2})$,

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- y_{int} will be close to the blowup solution of $z_t = \Delta z + |z|^{p-1}z + C$, with $(T-t)^{\frac{1}{p-1}} z(x,t) \sim f\left(\frac{x-a}{\sqrt{(T-t)|\log(T-t)|}}\right)$;
- y_{ext} will be close to the solution of $z_t = \Delta z + C$, with $y_{ext}(x,t) \sim y_{ext}(x,0)$, if $T>0$ is small.

Condition: F_{int} and F_{ext} should be bounded.

This is possible, if $T>0$ is small. Indeed, in that case, by construction, y_{int} will be

- unbounded *in* $B(a, \frac{\epsilon_2}{2})$,
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Since it happens that F_{int} and F_{ext} are supported in $\mathbb{R}^n \backslash B(a, \epsilon_2)$, they will be bounded.

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Rk. The "cut-off" strategy was efficient in the construction of a *periodic* solution to $z_t = \Delta z + |z|^{p-1}z$ in Mahmoudi-Nouaili-Zaag 2015.

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$$f(\eta) = \left(p-1+\frac{(p-1)^2}{4p}|\eta|^2\right)^{-\frac{1}{p-1}},$$

the construction is classical, from Bricmont-Kupiainen 1994, and also Merle-Zaag 1997.

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- The control of the positive part of the spectrum (finite dimensional) thanks to the degree theory (Brouwer's lemma).

Thank you for your attention!