EVOLUTIONARY DYNAMICS DO NOT LEAD TO EQUILIBRIA

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A major goal of evolutionary game theory is to clarify the connection between the outcome of simple adaptive processes, modelling the evolution of behavior in populations of boundedly rational agents, and equilibrium concepts. For a wide class of dynamics, if a solution converges to a point from an interior initial condition, then this point is a Nash equilibrium [1]; but, for any dynamics satisfying some minimal adaptivity and regularity conditions, there are games with a unique Nash equilibrium and such that, for an open set of initial conditions, the solution does not converge to the equilibrium but cycles [2].

There are several ways to try to find nonetheless a general connection between evolutionary dynamics and equilibria. A first possibility is to replace convergence to the set of Nash equilibria by some weaker connection, like convergence in time-average or simply a connection between strategies that survive and strategies that are played in equilibrium. Another possibility is to replace Nash equilibrium by a weaker concept. In light of the literature on adaptive heuristics [3], the main candidate would be correlated equilibrium [4].

In small dimension, some positive results can be obtained. For instance, in $3 \times 3$ symmetric games, under any convex monotonic dynamics [5], and for any interior initial condition, all strategies that do not belong to the support of any correlated equilibrium are eliminated [6]. However, we will show that, in higher dimension, no general connection can be found between strategies that survive under standard evolutionary dynamics and strategies in the support of correlated equilibria. Indeed, for a wide class of evolutionary dynamics, there exists an open set of games such that, for an open set of initial conditions, all strategies belonging to the support of at least one correlated equilibrium are eliminated; thus, only strategies that are not player in equilibrium survive.

To see this, we study $4 \times 4$ symmetric games built by adding a strategy to an outward-cycling Rock-Paper-Scissors game [7] and assuming some payoff inequalities. Under
the dynamics we consider, an attractor of the underlying Rock-Paper-Scissors game is asymptotically stable in the augmented, $4 \times 4$ game. This is in spite of the fact that, for an open set of such games, the unique strategy belonging to the support of a correlated equilibrium is the added, fourth strategy. The result follows. The proofs rely on Lyapunov functions and the stability condition for heteroclinic cycles given by Hofbauer [8].
For Nash equilibrium, the situation is even worse: elimination of all strategies in the support of at least one Nash equilibrium occurs for essentially all reasonable dynamics and, for the replicator dynamics and the best-response dynamics, it occurs in some games for almost all initial conditions. The games used are again variants of Rock-Paper-Scissors, and the proofs rely on Lyapunov functions, Poincaré sections, and the behavior of the time-average of solutions converging to an heteroclinic cycle [9].

Bibliographie