SOME RESULTS ON $\sigma$-CORES AND $\mathcal{G}$-CORES OF EXACT GAMES FOR LOCALLY COMPACT $\sigma$-COMPACT TOPOLOGICAL SPACES

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In a seminal paper Schmeidler (1972) [11] made an intensive study of the $\sigma$-core $C^\sigma(v)$ of exact games i.e. of the existence of $\sigma$-additive measures in the core of exact games. The aim of the present paper is to investigate the $\mathcal{G}$-cores of exact games defined on the set $\mathcal{B}$ of Borel sets of a locally compact and $\sigma$-compact topological space $\Omega$, so we deal with the existence of $\mathcal{G}$-continuous measures in the core of an exact game. Recall that a measure $P$ is said to be $\mathcal{G}$-continuous at $A \in \mathcal{B}$ if $\forall \{O_n\}_n \subset \mathcal{G}, O_n \uparrow \Omega :\liminf P(A \cap O_n) = P(A)$, and $\forall \{F_n\}_n \subset \mathcal{F}, F_n \downarrow \emptyset :\liminf P(A \cup F_n) = P(A)$ with $\mathcal{F}$ and $\mathcal{G}$ respectively the set of closed and open subsets of $\Omega$ and a measure $P$ is said to be $\mathcal{G}$-continuous $^1$ if it is $\mathcal{G}$-continuous at any set $A \in \mathcal{B}$.

Building on results of Y. Rébillé [10] which give natural decomposition à la Yosida-Hewitt of a finitely additive measure into a $\mathcal{G}$-continuous and a purely non $\mathcal{G}$-continuous part and with the help of the Vitali-Hahn-Saks theorem for charges (which can be found in Rao and Rao [9]), we show that the non-emptiness of the $\mathcal{G}$-core of $v$ noted $C^\mathcal{G}(v)$ is characterized by the simple property of $v$ being continuous from above at the empty set for closed sets. As a consequence the second conjecture of Schmeidler which asserts that "an exact game continuous at $\emptyset$ has a countably additive set function in its core" proves to be true on $(\mathcal{N}, \mathcal{P}(\mathcal{N}))$. We also obtain that every element in the core of $v$ is $\mathcal{G}$-continuous if and only if $v$ is continuous at $\Omega$ for open sets.

Then using techniques similar to Parker [8], we show that if moreover $\Omega$ is metrizable, then a finitely additive probability measure $P$ on $\mathcal{B}$ is $\sigma$-additive if and only

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$^1$-$\sigma$-continuity requires that convergence should hold for any monotone sequence $\{O_n\}_n, \{F_n\}_n$ in $\mathcal{B}$ and not solely in $\mathcal{G}, \mathcal{F}$, thus $\mathcal{G}$-continuity is a weaker property.
if $P$ is continuous at $\Omega$ for closed sets. We therefore deduce for such a topological space, by building upon Schmeidler (Theorem 3.2 [11]) that $C^s(v) = C(v)$ if and only if $v$ is continuous at $\Omega$ for closed sets.

**Bibliographie**


