Augmented Lagrangian preconditioner for Linear Stability Analysis of the incompressible Navier-Stokes equation

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Linear Stability Analysis is a widely used tool in the Fluid Mechanics community [1]. It consists in studying the stability of some stationary state of the governing equations by computing the spectrum of the linearized operator. On a computational point of view, the elementary brick of Linear Stability Analysis classically consists in inverting large sparse matrices arising from the spatial discretization of the linearized stationary Navier-Stokes equations. These inversions are required for the computation of both stationary state (e.g. with a Newton method) and eigenvalue spectrum (e.g. with Krylov-Schur + shift-invert). The mainly used strategy consists in inverting such systems by using direct solvers (MUMPS, UMFPACK, SUPERLU, etc). For large configurations (3D cases typically), the memory and cpu requirements might quickly become prohibitive. Thus, iterative methods, with adequate preconditioning, represent an interesting alternative. In the fluid stability community, a popular choice is to use time-stepper based approaches [2]. Very few studies, however, chose to tackle directly the inversion of the stationary jacobian. Indeed, whereas efficient preconditioning of the time-dependent Navier-Stokes equations is widely practiced, equivalent maturity for the stationary case is still to be achieved. In the present work, we propose to persevere in the second direction, by assessing the use of the recently proposed Augmented Lagrangian approaches for Linear Stability Analysis purposes [3],[4]. The practical parallel implementation in FreeFem++ through the PETSc/SLEPc interface will be discussed.

References

[1] V. Theofilis, «Global Linear Instability,» *Annual Review of Fluid Mechanics,* vol. 43, n° %11, pp. 319-352, 2011.

[2] L. S. Tuckerman, «Steady-state solving via stokes preconditioning; Recursion relations for elliptic operators,» chez *11th International Conference on Numerical Methods in Fluid Dynamics*, D. L. Dwoyer, M. Y. Hussaini et R. G. Voigt, Éds., Berlin, Heidelberg, Springer Berlin Heidelberg, 1989, pp. 573-577.

[3] M. Benzi, M. A. Olshanskii et Z. Wang, «Modified augmented Lagrangian preconditioners for the incompressible Navier-Stokes equations,» *International Journal for Numerical Methods in Fluids,* vol. 66, n° %14, pp. 486-508, 10 6 2011.

[4] T. Heister et G. Rapin, «Efficient augmented Lagrangian-type preconditioning for the Oseen problem using Grad-Div stabilization,» *International Journal for Numerical Methods in Fluids,* vol. 71, n° %11, pp. 118-134, 10 1 2013.