Solid-mechanics finite element simulations of the draping of fabrics: a sensitivity analysis

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Abstract

This paper covers numerical investigations of the draping of woven fabrics into a “hat” shape, combining a hemispherical cup with a wide flat rim. A mechanical approach is adopted using finite element analysis (FEA) methodology. In this, the fabric is considered as a solid sheet with mechanical properties and friction properties. In this study, a linear elastic anisotropic material model describes the deformation of fabrics. An explicit dynamic finite element analysis is applied and systematic parametric numerical studies are presented, which incorporate investigations of the effects of numerical parameters, material properties and processing conditions on the draping of fabrics. More specifically, the effects of the following variables and parameters are included: number of elements, number of time increments in the dynamic FEA analysis, punch speed, shear and tensile moduli of fabric, coefficient of friction for all interfaces and level of load on the fabric holder. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Woven fabrics are widely used in the manufacture of high performance composite products due to their balanced reinforcing properties, the ease of handling and their good processability. In many cases, the fabric architecture allows draping and shaping to occur over complex mould geometries, reducing the problem of discontinuities that may be encountered in unidirectional reinforcement.

In manufacturing processes such as resin transfer moulding (RTM), woven fabrics, with or without binder, are draped onto the mould surface. The prediction of the distortion of the fabric during draping and the changes in fibre orientation and fibre fraction are essential for the understanding of the manufacture process, the prediction of permeability of the fabric and the estimation of the mechanical properties of the composite products. Draping simulations of woven fabrics provide a way to analyse the deformation of the fabric and the simulation results can be used to predict local fibre orientations and fibre fractions.

There are two different approaches for the simulation of the draping of fabrics, the geometrical and the mechanical approach. The geometrical approach involves a type of mapping mostly based on a kinematic model. The fishnet model [1–7] is commonly used for the geometrical analysis of the draping of fabrics. Other geometrical techniques include the energy algorithm [8] and the optical projection method [2,9]. The effect of material properties is not generally included in the geometrical approach. The fishnet analysis, for example, treats the fabric as a pin-joined net of inextensible fibres, where the fibres are allowed to freely rotate at the crossovers without any slip.

The solid mechanics analysis of the draping of fabrics by using finite element methodology [10–13] takes into account the mechanical properties of the fabric and, hence, describes the physical process of draping. The fabric is considered as a continuum and the analysis of the draping of fabrics is essentially an extension of the deep drawing [14,15] or diaphragm forming simulation of metal forming processes [16]. The difference is that metal forming processes are associated with elasto-plastic material properties whereas the draping of fabrics in this study is related to anisotropic elastic properties.

The theoretical work in this study involves finite element analyses of the draping of woven fabrics having adopted the solid mechanics approach, in which the fabric is considered as a solid continuum with anisotropic, elastic properties and friction properties. Draping studies cover the model
geometry of a hemispherical cup, combined with a surrounding flat rim. The explicit dynamic finite element analysis is employed. The scope of this study is to carry out numerical investigations about the effects of material properties and processing conditions on the draping of fabrics into the aforementioned geometry, as well as the effects of certain numerical parameters on the draping simulations. These extensive parametric studies and sensitivity analyses of draping are possible by adopting the mechanical approach in the modelling of the draping of fabrics.

2. Mathematical and numerical modelling

A dynamic mechanical analysis has been applied to the problem of draping of fabrics based on the Navier’s equations of motion [17]:

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla \mathbf{\sigma} + \mathbf{f}$$

(1)

where \(\rho\) is the density, \(t\) is time, \(\mathbf{\sigma}\) is the stress tensor, \(\mathbf{f}\) is the body force vector and \(\mathbf{u}\) is the velocity of a particle following the fabric deformation where

$$\dot{\mathbf{u}} = \frac{d\mathbf{x}}{dt}$$

(2)

and \(\mathbf{x}\) is the vector of coordinates of a particle in the Lagrangian frame of reference (a frame of coordinates following the motion of the particle and the deformation of fabric).

The fabric is assumed to be a solid continuum with linear elastic, anisotropic mechanical properties described by the constitutive set of equations:

$$\mathbf{\sigma} = \mathbf{Q}\epsilon$$

(3)

where \(\epsilon\) is the strain tensor and \(\mathbf{Q}\) is the modulus matrix. Other assumptions concerning the woven fabric include: (a) that it should have a plane of symmetry coinciding with the median plane of the fabric; and (b) the thickness of the fabric should be small enough, so that the plane stress approach can be used.

During the draping of fabrics, as the fabric comes into contact with the mould, the contact analysis is based on a Coulomb friction model which assumes that no relative motion occurs if the equivalent friction stress is less than a critical stress, \(\sigma_{\text{crit}}\), whereas full slip occurs if the equivalent friction stress exceeds \(\sigma_{\text{crit}}\), where

$$\sigma_{\text{crit}} = \mu \sigma_n$$

(4)

with \(\mu\) being the friction coefficient and \(\sigma_n\) the stress normal to the contact plane. Here \(\mu\) may vary between 0 and 1, with \(\mu = 0\) indicating full slip with no friction and \(\mu = 1\) indicating sticking friction.

The finite element code ABAQUS/Explicit v5.6 [18] is used for the numerical simulations. The mechanical model is solved by applying the finite element numerical technique with an explicit time discretisation scheme. This scheme obtains values of time dependent variables at \(t + \Delta t\) based entirely on available values at time \(t\). The explicit dynamics analysis procedure is based upon the implementation of an explicit central difference integration rule where

$$\mathbf{u}^{(i+1/2)} = \mathbf{u}^{(i-1/2)} + \frac{\Delta t (i+1)}{2} \mathbf{u}^{(i)}$$

(5)

$$\mathbf{u}^{(i+1)} = \mathbf{u}^{(i)} + \Delta t (i+1) \mathbf{u}^{(i+1/2)}$$

(6)

where \(\mathbf{u}\) and \(\dot{\mathbf{u}}\) are the velocity and acceleration vectors, respectively. The superscript \((i)\) refers to the time increment number and \(i - 1/2\) and \(i + 1/2\) refer to mid-increment values.

By discretising Eq. (1) in a finite element domain and following the explicit time discretisation technique, the acceleration vector is given by:

$$\ddot{\mathbf{u}}^{(i)} = \mathbf{M}^{-1} (\mathbf{A}^{(i)} - \mathbf{I}^{(i)})$$

(7)

where \(\mathbf{M}\) is the diagonal or “lumped” element mass matrix, \(\mathbf{A}\) is the applied load vector, and \(\mathbf{I}\) is the internal force vector. The advantage of the explicit time procedure is that it requires no iterations and no tangent stiffness matrix.

The numerical procedure involves the explicit calculation of the current accelerations, velocities and displacements of each element node at every new time-step. This leads to the calculations of the current strains, stresses and forces, the values of which will be used in the explicit calculations at the subsequent time-step.

The central difference operator, which is the most commonly used explicit operator for stress analysis applications, is only conditionally stable. The stability limit is approximately equal to the time for an elastic wave to cross the smallest element dimension in the model, which is given by the following relation:

$$\Delta t = \min \left( \frac{L_e}{c} \right)$$

(8)

where \(L_e\) is a characteristic element dimension and \(c\) is the propagation speed of the elastic wave. This yields very small time-steps in an explicit dynamic analysis, which would result in extremely lengthy computations of the real draping process.

In order to accelerate the numerical computations, the forming process is simulated at artificially higher draping rates (increased punch speed) which increases artificially the inertia effects in Eq. (7) and might generate high frequency numerical oscillations. To control these oscillations, a small amount of artificial damping is introduced in the dynamic analysis in the ABAQUS code in the form of an artificial bulk viscosity. Its purpose is as a numerical effect only to improve the modelling of high speed dynamic events, therefore the bulk viscosity pressure is not included in the material point stresses, in another words, it is not considered as a part of the material’s constitutive response.
3. Geometrical finite element model and basic set up of draping simulations

The draping simulations following the mechanical approach are performed using the FEA computer code ABAQUS/Explicit v5.6. The geometrical model, shown in Fig. 1, includes a rigid punch, a die and a holder and the woven fabric blank.

The punch is a hemispherical male mould with a radius of 98.6 mm joined with a cylindrical flange at the upper end. The die is a hemispherical hat shape female mould with a radius of 100.4 mm in the hemisphere and a radius of 240 mm in the flat disc part. The holder is a flat ring to hold the fabric during draping. The blank material is a woven fabric of thickness of 0.64 mm representing an eight harness satin glass woven fabric.

Four-node three-dimensional R3D4 rigid surface elements are used to model the punch, die and holder. Four-node shell elements SR4 are used to model the woven fabric blank. This type of element is of bilinear finite strain and is designed to handle large deformations and large rotations. The initial shape of elements is chosen to be squares. The material directions of the undeformed woven fabric coincide with the sides of the initial mesh so that the finite element mesh is used to represent the fibre yarns in the woven fabric blank. The fabric blank has a mesh of 2500 shell elements (50 × 50) in most parametric studies. At this stage, the finite element analysis applied in this study does not take into account the effect of the change of material directions during draping on the constitutive model. This has been changed in a second study by the authors [19].

The draping is simulated by an explicit dynamic analysis. In the corresponding experimental set up available in our Laboratory of Composites Processing, the male half of the mould is made from aluminium and the female half of the mould is made from “perspex”. In the simulations, friction is considered between the interfaces of punch/fabric, die/fabric and holder/fabric. A mass of 0.6396 kg is attached to the holder and a concentrated load of 22.87 kN is applied to the control node of the holder. The holder is then allowed to move only in the vertical direction to accommodate the gap variation available to the woven fabric during draping.

The fabric properties used in a typical numerical set up are:

- In-plane shear modulus $G_{12} = 7.8 \times 10^4$ Pa [20],
- Tensile moduli $E_{11} = E_{22} = 3.5 \times 10^{10}$ Pa,
- Poisson’s ratio of 0.1.

The friction coefficients at the different interfaces have been taken as follows (see Appendix A and Table 1):

\[
\begin{align*}
\mu_{\text{punch/fabric}} &= 0.2, \\
\mu_{\text{die/fabric}} &= 0.5, \\
\mu_{\text{holder/fabric}} &= 0.
\end{align*}
\]

In the draping simulation, as seen in the Fig. 2, the punch moves down at a constant speed, typically 30 m/s, and the whole process lasts 1000 to 2000 time-steps or hundreds of seconds in terms of CPU time.

4. Parametric studies: results and discussion

Fig. 2 presents an example of the fabric deformation at different times during an explicit dynamic, finite element, solid mechanics analysis of draping. Such an analysis gives us the opportunity to observe the different stages of deformation in a stepwise manner and to detect the start and source of instabilities, wrinkles and other potential problems.
in draping. However, it is necessary to point out that there are many factors influencing the predictions from a computational simulation of draping. Parametric studies are essential to aid validation of the numerical simulations, to establish the critical level of key numerical parameters, to investigate the necessity for the provision of accurate experimental data for material properties, to compare different materials and to provide a first set of data for the design of moulding tools. The factors investigated in this study include numerical parameters such as the punch speed, the mesh size and the number of time increments, material properties such as the tensile and shear modulus of the fabric and the friction coefficients, and processing conditions such as the force acting on the holder in the draping simulations.

4.1. Effect of punch speed

The computer time involved in running the draping simulation using explicit time integration with a given mesh is directly proportional to the time period of the event. In explicit dynamic analyses, the stable time increment size is a function of the mesh size and the material stiffness. In order to run the simulation efficiently without sacrifice of precision, it is usually desirable to run the simulation at an artificially high speed compared to the real draping process. Investigations were carried out for different punch speeds, \( v \), from 0.1 to 100 m/s. The shapes of the deformed mesh in Fig. 3 show that different punch speeds have no visible effect on the deformed shape of the fabric. This might indicate that, for this range of speeds and the selected damping in the numerical procedure, the inertial forces will not play a dominating role for forming rates higher than 0.1 m/s.

Fig. 4(a) displays the predicted shear angle along the diagonal line of the fabric (from the centre to the corner of the fabric piece) against the normalised line distance \( \frac{L}{S} \), where \( L \) is the arc distance from the apex of the hemispherical mould and \( S \) is the equivalent quarter arc length of the sphere. \( L/S = 0 \) is located at the apex which is not sheared due to symmetry. The maximum shear is encountered at \( L/S = 1 \), where the hemispherical shape ends and the flat rim starts. The most irregular solution is given by a punch speed \( v = 100 \) m/s, departing from the other solutions especially at \( L/S = 1 \). The low punch speed of 0.1 m/s yields the smallest oscillations at \( L/S > 1.5 \) which lie in the flat part of the mould where there is an increased possibility for wrinkling in real draping as well.

Fig. 4(b) shows that the predictions of the maximum
Fig. 3. Fabric deformation at different punch speeds, fabric mesh = 50 x 50.
shear angle around $L/S = 1$ are similar for the examined punch speeds, except at the speed of 100 m/s where the solution is influenced by inertial effects resulting in large oscillations as also presented in Fig. 4(a). Fig. 4(c) illustrates that the computational costs range from $100 \times 10^3$ cp (s) to $100 \times 10^3$ cp (s) for a corresponding range of punch speeds from 100 to 0.1 m/s. As a result, a punch speed around 30 m/s has been considered acceptable for the balance between the accuracy and the computation time (see Fig. 4(b) and (c)). Similar punch speeds have also
been found suitable in the explicit dynamic analysis of metal forming [21,22].

4.2. Effect of mesh size

To assess the effect of mesh size, different mesh sizes were employed for the fabric blank, namely 10 × 10, 30 × 30, 50 × 50, 70 × 70 and 100 × 100 shell elements. The predictions of shear angle of the draped woven fabric along the diagonal direction are shown in Fig. 5(a) and the deformed shapes are included in Fig. 6. The predictions with the 10 × 10 fabric mesh seem to be much less accurate whereas the accuracy is much improved for meshes finer than 30 × 30 elements. From the results presented in

![Graphs showing the effect of mesh size on predictions of shear angle and CPU time](image-url)
Fig. 5(b), the precision regarding the maximum shear angle along the diagonal line converges with increasing number of elements while the computation time experiences a logarithmic rise (see Fig. 5(c)). Apart from the need of more computation time due to the larger number of elements, fine meshes require smaller stable time increments, thus adding to the total number of time increments and the computation time requirements. A mesh of $50 \times 50$ elements appears to be a good compromise for the purpose of parametric studies.

4.3. Effect of time-step

Another effect on the performance of the explicit dynamic FEA analysis is the time increment or the time-step in the draping simulation. Normally, a stable time-step is automatically calculated by the computer code ABAQUS Explicit 5.6 based on the size of a typical element and the wave propagation speed. Nevertheless, it was thought that in some cases intervention might be needed. The effect of the time increment was studied by using different number of increments for a run based on a mesh size of $50 \times 50$ elements and a punch speed at 30 m/s. Fig. 7(a) and (b) show that the automatic step (inc. = 1260) chosen by ABAQUS is valid for parametric studies, although again in some cases increasing the number of time-steps (smaller time-steps) would result in higher accuracy in the predictions. However, the required computation time increases logarithmically with the number of time increments (see Fig. 7(c)).

4.4. Effect of the elastic moduli of fabric

The investigated mechanical properties of fabrics include the shear and tensile moduli, $G_{12}$ and $E_{11}$, respectively. Shear deformation is the dominant mode in the draping of fabrics. Given that experimental shear stiffness data for dry woven fabrics may not be highly accurate (due to the very low values of shear modulus), a sensitivity study has been conducted by varying the value of shear modulus for the woven fabric through the large range of $7 \times 10^5$–$7 \times 10^7$ Pa. Fig. 8 illustrates that the large scale change in shear modulus, $G_{12}$, has very little effect on the shearing of the fabric in the hemisphere ($L/S < 1$) but higher $G_{12}$ values alter the shear angles on the flat surface. This suggests that an accurate value of shear modulus in the low range of values may not be important as long as the ratio of $G_{12}/E_{11}$ remains very small (the ratio is 0.01 for the upper bound value of $G_{12}$ in this study). Some oscillations in the
shear angle around $L/S = 1$ at $G_{12} = 780$ Pa suggest the sensitivity of the numerical solution at extremely low moduli.

The tensile modulus $E_{11} = E_{22}$ was varied in the range of $7 \times 10^6 - 3.5 \times 10^{10}$ Pa while the shear modulus was retained at $7.8 \times 10^4$ Pa; the results are presented in Figs. 9 and 10. Fig. 10(a) illustrates that a very low tensile modulus, more suitable for rubbery materials, would lead to stretching the central core of the blank into the hemisphere and leave the flat part unaffected. In practice, fabric yarns might exhibit a rubbery material tensile modulus only at very low strains [10] (less than 0.1%) while the yarn has still some crimp. Otherwise, they generally exhibit a high tensile modulus at
the upper end of the examined range, for which there are no great differences in the numerical solutions, especially in the hemispherical part of the mould.

4.5. Effect of friction at interfaces

Coefficients of friction are included at the interfaces of punch/fabric, die/fabric and holder/fabric. However, to simplify the problem, the coefficient of friction value is assigned to zero at the interface of holder/fabric. At the interfaces of punch/fabric and die/fabric, the value of the coefficient of friction changes in the range of 0.0–0.5. Fig. 11(a) displays the results of the shear angle along the diagonal line of the fabric when it has been taken that \( \mu_{\text{punch/fabric}} = 0.25 \) and \( \mu_{\text{die/fabric}} \) varies in the range of 0.0–0.5. Fig. 11(b) displays the results of the shear angle along the diagonal line of the fabric when it has been taken that \( \mu_{\text{punch/fabric}} = 0.5 \) and \( \mu_{\text{die/fabric}} \) varies in the range of 0.0–0.5. Both figures show some differences in the exact location of the maximum shear angle and differences on the flat part of the mould. In general from past experience in mechanical engineering applications, the introduction of some friction into the mechanical forming may be sometimes critical. Appendix A includes experimental data of the static friction coefficient at various types of interfaces including glass fibre, plain and satin woven fabrics, and aluminium and “perspex” as mould materials. At this stage, it must also be mentioned that in the experimental set up available
Fig. 10. The deformed fabric shapes with different values of $E_{11}$ ($G_{12} = 7.8 \times 10^4 \text{ Pa}$).

(a) $E_{11} = E_{22} = 7.0 \times 10^6 \text{ Pa}$

(b) $E_{11} = E_{22} = 7.0 \times 10^7 \text{ Pa}$

(c) $E_{11} = E_{22} = 7.0 \times 10^9 \text{ Pa}$
in our Laboratory of Composites Processing the punch is made from aluminium and the die is made from “perspex”.

4.6. Effect of the force acting on the holder

The effects of the load on the holder are demonstrated in Fig. 12. For relatively small and medium loads the solutions in the hemispherical cup are generally similar whereas there are some variations on the flat part. However, when the force is raised to 0.23 MN the motion of the fabric is severely restricted and the shear angle is associated with frequent and large spatial variations which also indicate instabilities in the numerical solution.

5. Conclusions

This study presented mathematical modelling of the
draping of fabrics on the basis of a mechanical approach where the fabric was considered a solid continuum with linear elastic anisotropic properties and friction at the interfaces. Simulations were carried out by employing an explicit dynamic finite element analysis. An extensive numerical sensitivity analysis was carried out for the draping of a fabric into the double curvature geometry of a hemispherical hat with a wide flat rim.

The analysis was first investigated in terms of the numerical parameters, namely the punch size, mesh size and time-step. It was decided that, in order to accelerate computations, it is possible to artificially increase the rate of draping to a punch speed of 30 m/s with the aid of numerical damping without introducing significant numerical inertia effects. At this speed and for the studied model geometry, a 50 × 50 mesh for the fabric blank and 1260 time-steps were considered suitable in terms of numerical stability and accuracy and for the purpose of parametric studies.

Variations of the in-plane shear modulus of fabric in the range of $7.8 \times 10^3$–$7.8 \times 10^5$ Pa had little effects on the draping predictions. However, variations of the tensile modulus had significant effects: A tensile modulus for rubbery materials would cause deformation only in the central core of the fabric whereas variations at the upper bound of the values of tensile modulus, more suitable for glass and carbon, had no effects on the numerical solution and resulted in deformation throughout the whole fabric area during draping.

On the other hand, friction at the interfaces of die/fabric and punch/fabric had some effects especially on the predicted shear angles at the flat part of the mould. Friction at the punch/fabric interface in the range of 0.25–0.5 decreased spatial oscillations and led to solution stability whereas an optimum friction coefficient at the die/fabric interface could be identified around 0.25; high values of $\mu_{\text{die/fabric}}$ led to significant spatial oscillations of the shear angle at the flat part of the mould.

The final parameter to be investigated was the load on the holder. Very large loads of the order of 0.2 MN restricted severely the motion of fabric and caused spatial oscillations in the shear angle throughout the whole fabric area during draping. Suitable values of force on the holder were in the range of 22–2200 N although there were some differences in the numerical solution at the flat part of the mould within this range.

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Appendix A. Experimental measurement of the coefficient of friction

Measurements of the static friction coefficient were carried out following the ASTM standards [23,24] as illustrated in Fig. 13. This involves the measurement of the friction coefficient at the interface between the upper surface of the inclining plate and the bottom surface of
the block standing on it. The inclining plate was made from aluminium. Two alternative blocks were used, made from aluminium and “perspex”, respectively. In order to test the surface of a fabric, either the aluminium block or the upper surface of the inclining plate, or both, were covered with the fabric.

The coefficient of friction was determined by raising the surface gradually until the block started to move. At that point the angle of the inclined surface was recorded by using an inclinometer and the coefficient of friction was evaluated by the relation:

$$\mu = \tan \alpha$$  \hspace{1cm} (A1)

The following glass fibre fabrics were tested: (a) an eight harness satin Y0736 supplied by Fothergill Ltd. and (b) a plain weave Y0212 also supplied by Fothergill Ltd.

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