Title: Convex relaxation for a class of free boundary problems

Abstract: This lecture is based on a duality scheme for non convex variational problems which has been developed in collaboration with I Fragala (2016 Arxiv) and Minh Pahn (Toulon). Typically this theory concerns minimum problems of the kind

$$\inf\left\{\int_\Omega (|\nabla u|^p + g(u))\, dx \quad : \quad u \in W^{1,p}(\Omega)\ ,\ u = u_0 \text{ on } \partial\Omega\right\}$$

where $g : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$ is a non convex l.s.c function with possibly many jumps. In this talk, I will report on two recent related results specific to the case $p = 1$:

- the first one concerns an exclusion principle which states that minimizers take values outside the set $\{g^{**} < g\}$. This principle allows convex relaxation and then we focus on a multiphase problem that we treat numerically by means of a primal-dual algorithm.

- the second one concerns a variant of the Cheeger problem in a convex subset $D \subset \mathbb{R}^2$ for which we construct explicit calibrating fields. This is done by using a locally Lipschiz potential whose trace on ∂D coincides with the normal distance to the cut-locus.