

Transport equations in biology- Missprints

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Abstract

This file presents a list of missprints and improvements in the Lecture Note, Birkhauser 2007.

1 Chapter 1, Section 1.2.3, Proof of Lemma 1.3

In order to prove Lemma 1.3, it is easier to use the Lyapunov functional:

$$\begin{aligned} \frac{d}{dt} \left[-\bar{n} \ln n(t) - \sum_i \bar{S}_i \ln S_i(t) + n(t) + \sum_i S_i(t) \right] \\ = -R \sum_i \frac{S_{0i}}{S_i(t)\bar{S}_i} |S_i(t) - \bar{S}_i|^2 \end{aligned}$$

Because $H(t) = -\bar{n} \ln n(t) - \sum_i \bar{S}_i \ln S_i(t) + n(t) + \sum_i S_i(t)$ is bounded from below and because $S_i(t)$ is bounded (use for instance the quantity $M(t)$) we conclude that

$$\int_0^\infty \sum_i \frac{S_{0i}}{S_i(t)\bar{S}_i} |S_i(t) - \bar{S}_i|^2 < \infty,$$

$$\int_0^\infty \sum_i |S_i(t) - \bar{S}_i|^2 < \infty.$$

Using that $S_i(t)$ is Lipschitz continuous, this implies that $S_i(t) - \bar{S}_i \rightarrow 0$ as $t \rightarrow \infty$. \square

This argument also works for multiple species and multiple nutrients, see [1]

2 Chapter 2

Proof of Theorem 2.2 : r stands for $\varrho(t)$

Line before (2.18) it should be: $[Q - P](\varrho(t))$ has a limit

3 Chapter 3

Section 3.9.1 is badly written and the condition is uncorrect. Here is a new version

3.9.1 Renewal equation for cell division cycle (one phase)

This section presents some improvement of the arguments in section §3.7 to obtain exponential time decay. We show that it is possible to prove an explicit exponential rate of convergence, in the natural norm, for situations more general than the mere assumption (3.18) We illustrate the idea on a model for cell division cycle with a single phase, see section §3.9.5 for a motivation. This is special case of age structured equation written as

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} n(t, x) + \frac{\partial}{\partial x} n(t, x) + k(x) n(t, x) = 0, \quad t \geq 0, x \geq 0, \\ n(t, x = 0) = 2 \int k(y) n(t, y) dy, \\ n(t = 0, x) = n^0(x), \end{array} \right. \quad (1)$$

where the function $k(\cdot)$ can be interpreted as a mitosis rate. When cells undergo mitosis, they are withdrawn from the balance equation at age x with a rate $k(x)$ and create two daughter cells at age $x = 0$ with the same rate.

For our theoretical study, we take for simplicity the mitosis rate

$$k(x) = \beta \mathbf{1}_{\{x \geq x_*\}}, \quad \beta > 0, \quad x_* > 0. \quad (2)$$

We recall here the definitions of the eigenlements, and readily check that they exist here because condition (3.22) is fulfilled,

$$\left\{ \begin{array}{l} \frac{\partial}{\partial x} N(x) + (k(x) + \lambda_0) N(x) = 0, \quad x \geq 0, \\ N(x = 0) = 2 \int k(y) N(y) dy, \quad N > 0, \quad N(0) = 1, \\ \frac{\partial}{\partial x} \phi(x) - (k(x) + \lambda_0) \phi(x) = -2\phi(0)k(x), \quad x \geq 0, \\ \phi \geq 0, \quad \phi(0) = 1. \end{array} \right.$$

Because k vanishes close to 0, assumption (3.18) does not hold here. Nevertheless we can obtain an exponential rate of convergence

Theorem 3.1 *We assume (2) and*

$$2\beta x_* e^{-\frac{\beta x_*}{1+2\beta x_*}} < 1, \quad (3)$$

then, for some positive function $\bar{\phi}$ and real number μ_0 given below, we have

$$\int |\tilde{n}(t, x) - m^0 N(x)| \bar{\phi}(x) dx \leq e^{-\mu_0 t} \int |n^0(x) - m^0 N(x)| \bar{\phi}(x) dx, \quad (4)$$

with $m^0 = \int n^0 \phi$ and some $\mu_0 > 0$ given in the proof below.

Proof. *First step.* We compute the eigenvalue λ_0 . The basic functions N , $Q = N\phi$ are given in this case by

$$N(x) = \begin{cases} e^{-\lambda_0 x} & \text{for } x \leq x_*, \\ e^{-(\lambda_0 + \beta)x} e^{\beta x_*} & \text{for } x \geq x_*, \end{cases}$$

$$Q(x) = \begin{cases} 1 & \text{for } x \leq x_*, \\ 2 \frac{\beta}{\lambda_0 + \beta} N(x) & \text{for } x \geq x_*. \end{cases}$$

Especially, λ_0 is defined by $2 \int k(x)N(x)dx = 1$ which gives

$$\lambda_0 + \beta = 2\beta e^{-\lambda_0 x_*}, \quad (5)$$

an equation which always has a unique solution and $\frac{\beta}{1+2\beta x_*} < \lambda_0 < \beta$.

Second step. We consider a function $c(x)$ to be chosen later (that replaces $2k(x)$ in the right hand side of the equation defining ϕ) such that

$$c(x) \geq 0, \quad \int c(x)N(x)dx = 1. \quad (6)$$

And we set

$$\bar{Q}(x) = 1 - \int_0^x c(y)N(y)dy, \quad \bar{Q} = N\bar{\phi}. \quad (7)$$

As in the proof of Theorem 3.5, we consider the function $h = n(t, x)e^{-\lambda_0 t} - m^0 N(x)$, which still satisfies equation (1) (with a different Cauchy data) and $\int h(t, x)\phi(x)dx = 0$. We have by a simple combination of the equation on h and the equation

$$\begin{aligned} \frac{\partial}{\partial x} \bar{\phi}(x) - k(x) \bar{\phi}(x) &= -c(x), \\ \frac{d}{dt} \int |h(t, x)| \bar{\phi}(x) dx &= - \int |h(t, x)| c(x) dx + \int h(t, x) 2k(x) dx \\ &= - \int |h(t, x)| c(x) dx + \int h(t, x) [2k(x) - \mu_0 \phi(x)] dx \\ &\leq - \int |h(t, x)| c(x) dx + \int |h(t, x)| |2k(x) - \mu_0 \phi(x)| dx. \end{aligned}$$

In order to choose $c(x) = \mu_0 \bar{\phi}(x) + |2k(x) - \mu_0 \phi(x)|$, for some $\mu_0 > 0$, and since c is only constrained via (6), it is enough to be sure that

$$\int |2k(x) - \mu_0 \phi(x)| N(x) dx < 1.$$

We assume that μ_0 is small enough so that $2k(x) > \mu_0 \phi(x)$ for $x > x_*$, then we have

$$\int |2k(x) - \mu_0 \phi(x)| N(x) dx = \int_0^{x_*} \mu_0 \phi(x) N(x) dx + \int_{x_*}^{\infty} [2\beta - \mu_0 \phi(x)] N(x) dx < 1.$$

This is also written

$$\int_0^{x_*} \phi(x) N(x) dx - \int_{x_*}^{\infty} \phi(x) N(x) dx < 0, \quad \text{or also} \quad x_* - \frac{1}{\lambda_0 + \beta} < 0,$$

which itself is equivalent to

$$2\beta x_* e^{-\lambda_0 x_*} < 1,$$

and is satisfied under (3) thanks to the lower bound on λ_0 .

4 Chapter 4

Theorem 4.6: b_m stands for B_m .

5 Index

index pages are sometimes shifted from two pages

References

- [1] S. Mirrahimi, B. Perthame and J. Y. Wakano, Evolution of species trait through resource competition. Rapport de recherche: hal-00566888.