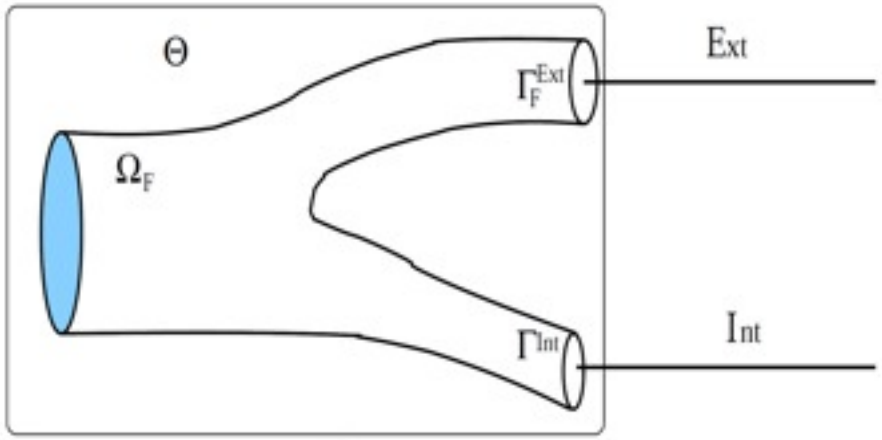
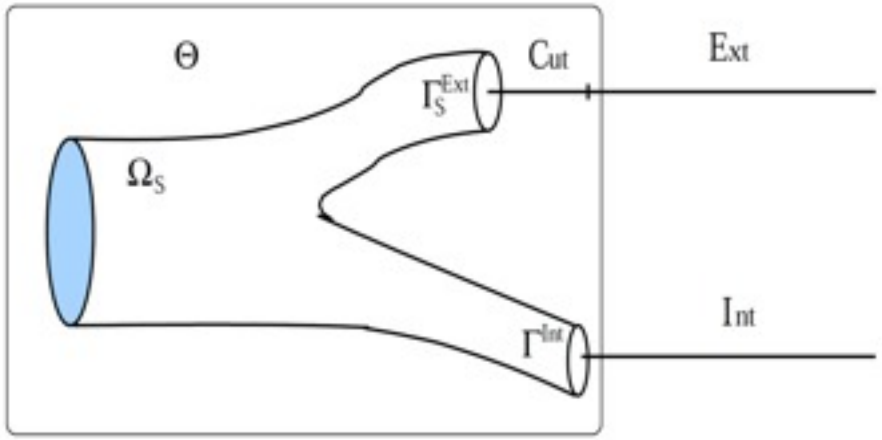
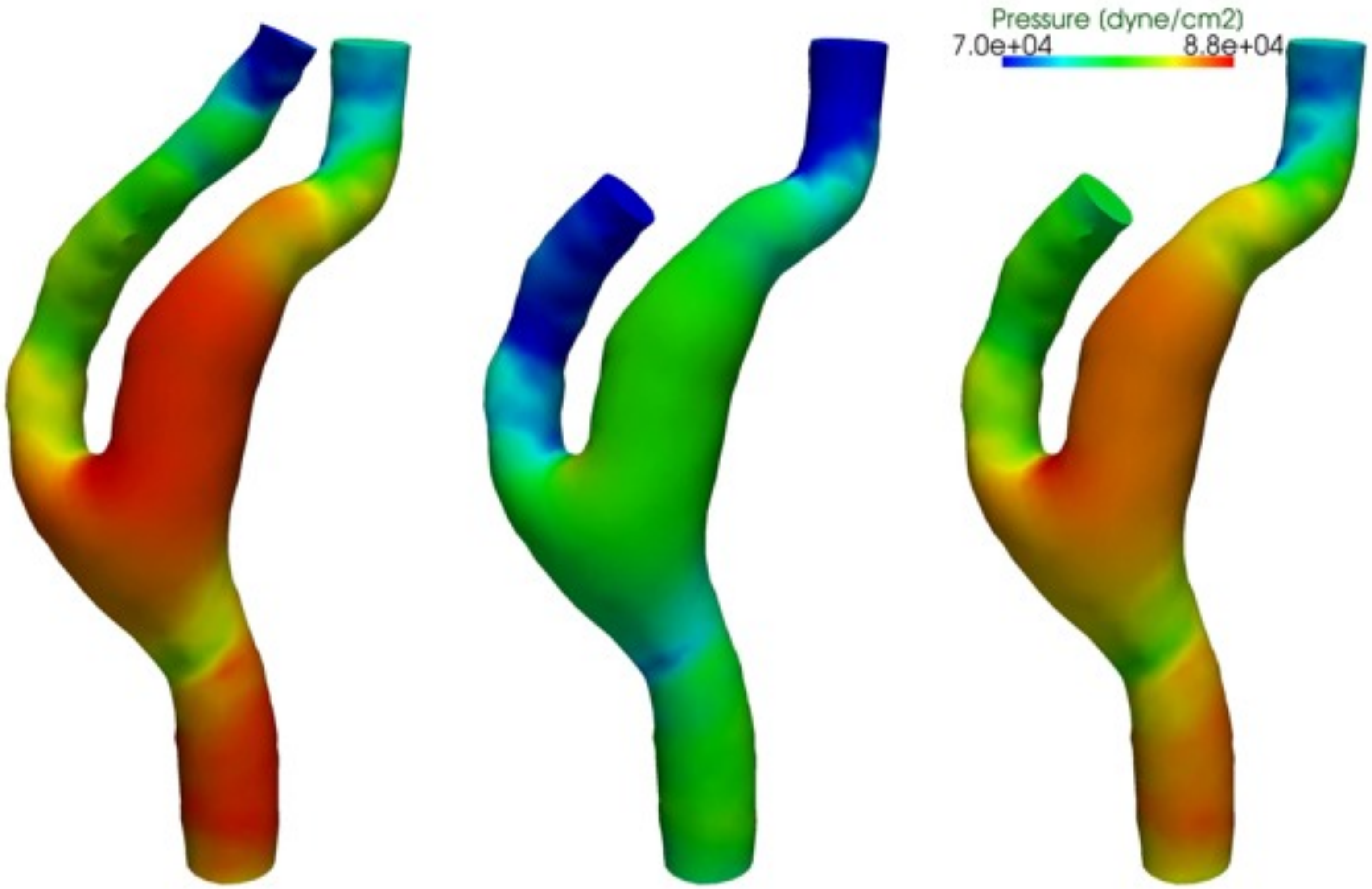
Comparison among different models





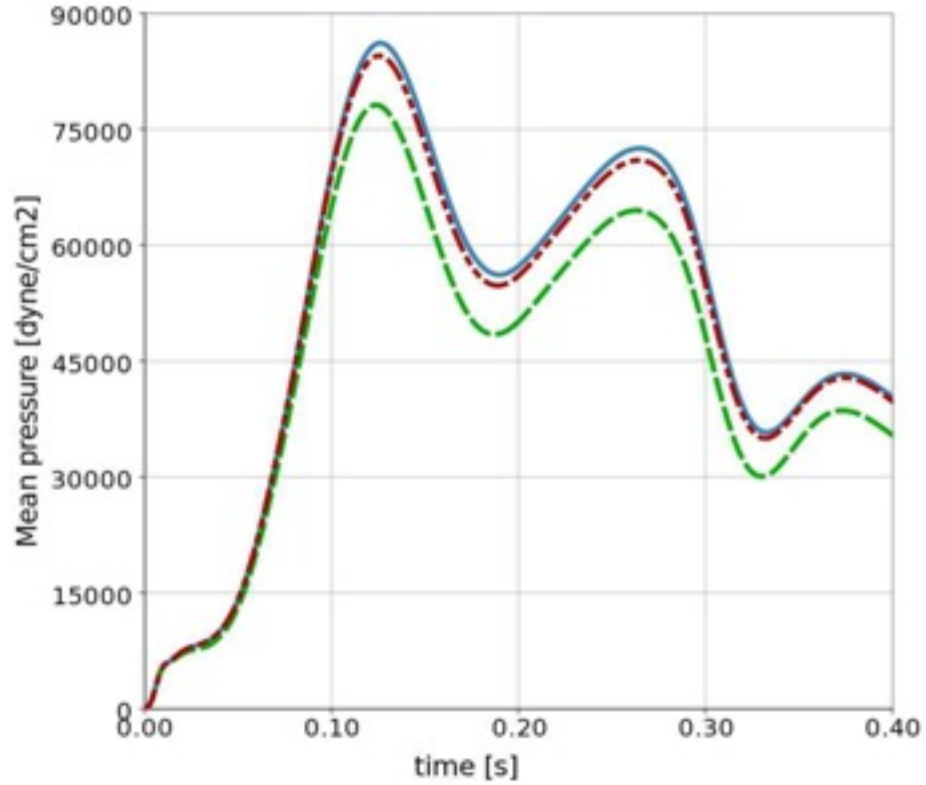
Full 3D/FSI model **3D/FSI-1D model** **3D/FSI-1D model with tapering**

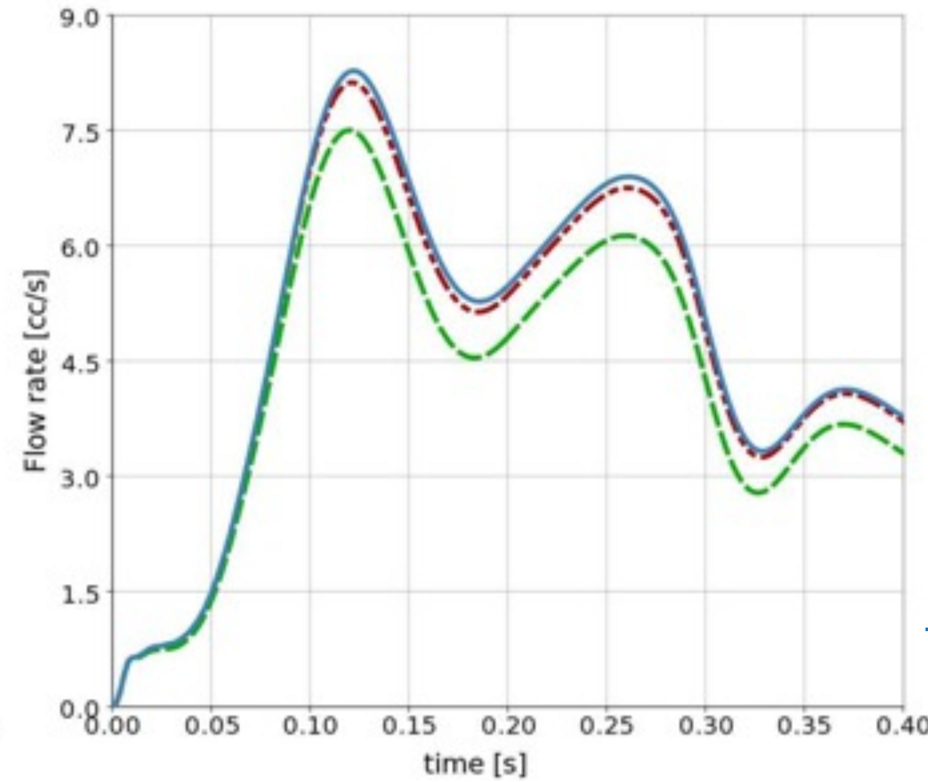


Good agreement between the simplified model with tapering in the 1D model and the full model

Comparison among different models

Mean pressure and flow rate at the internal carotid (up) and at the external carotid (bottom)

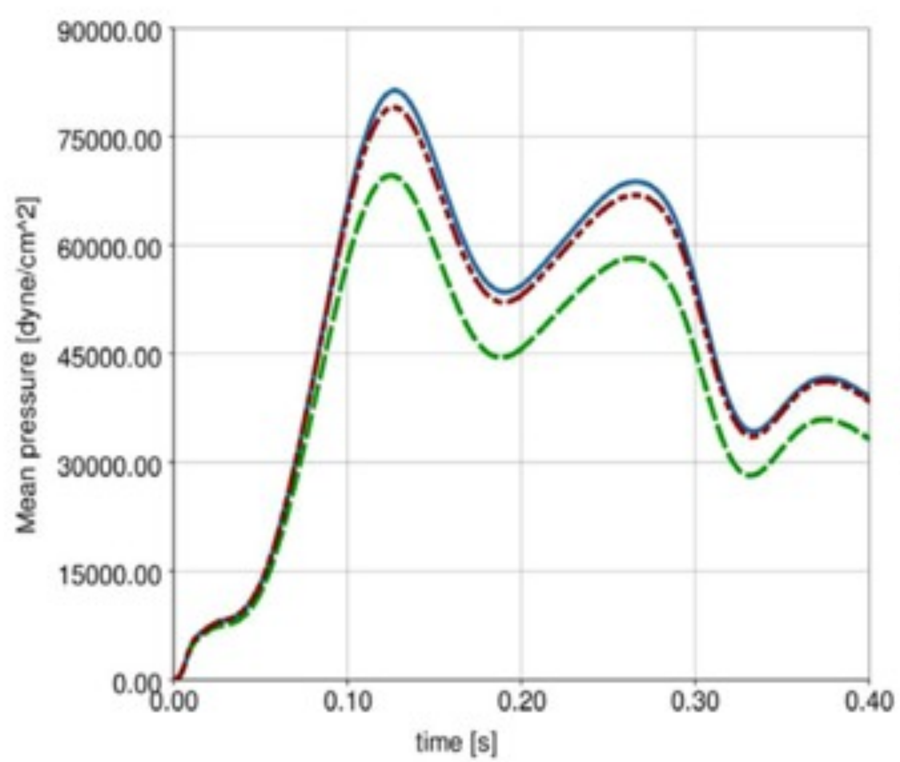


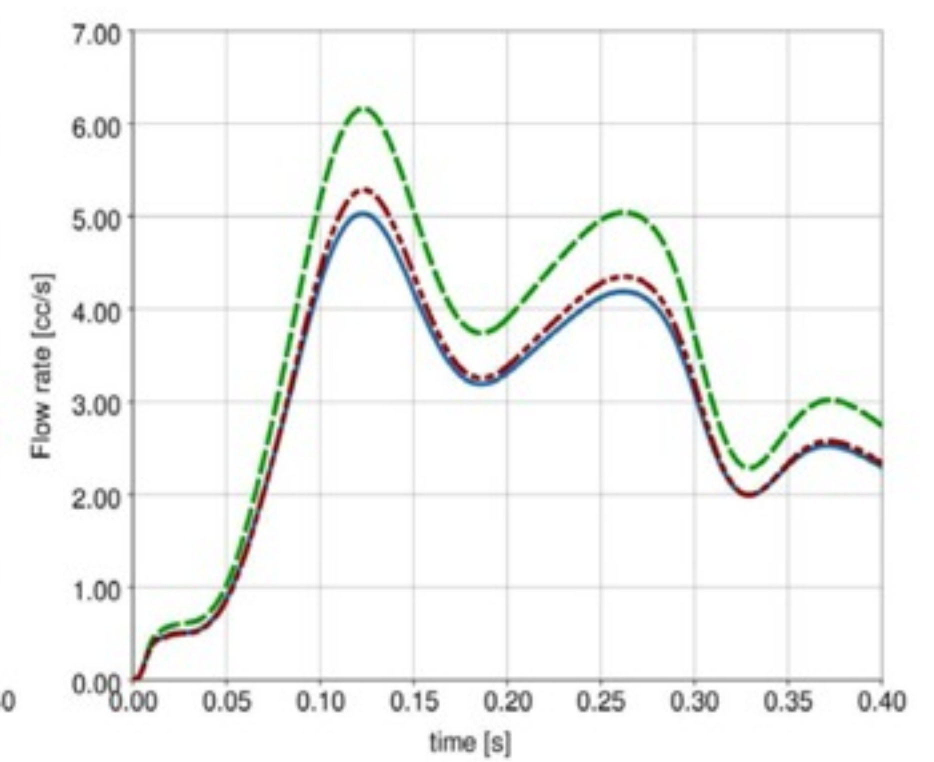


- - - - 3D/FSI-1D model no tapering

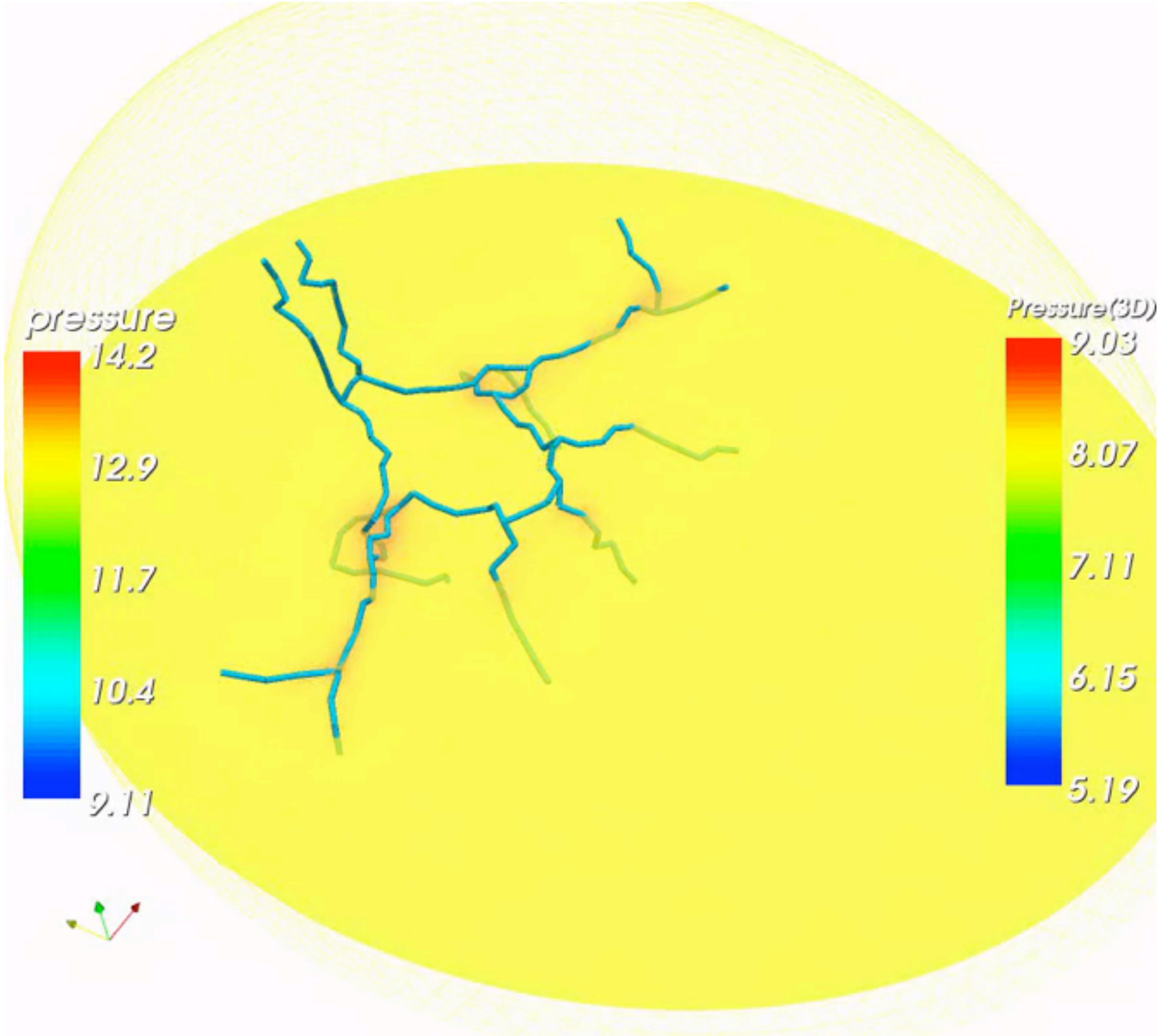
- - - - 3D/FSI-1D model with tapering

—— Full 3D/FSI model





Blood Flow and Oxygen Transport in the Brain



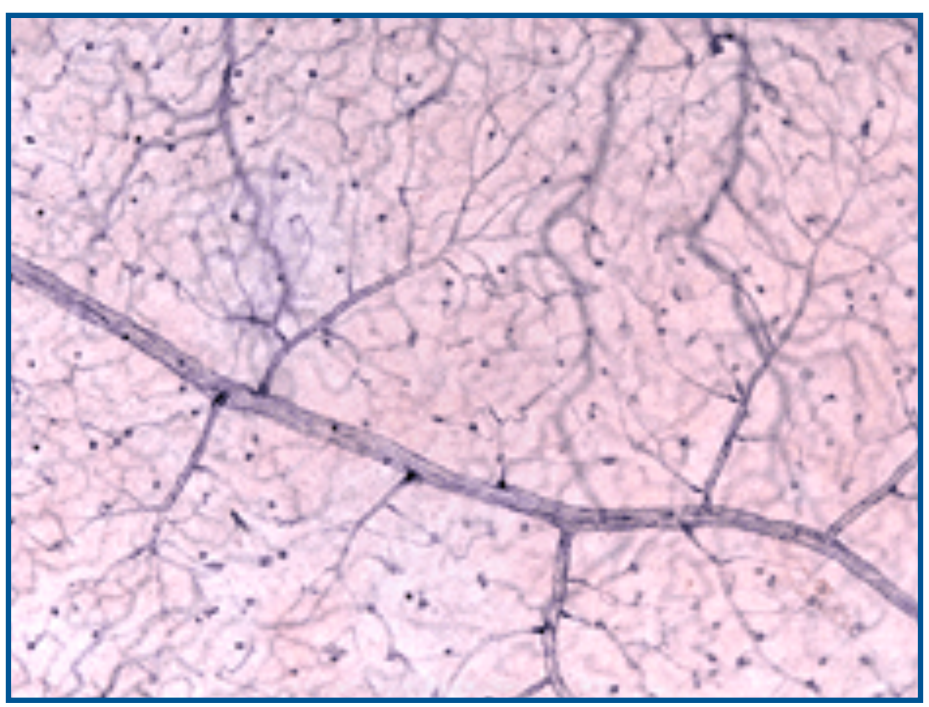
1D (vessels) and
3D (brain tissue)
blood pressures
with pulsatile input blood
flow rate and left carotid
artery occlusion

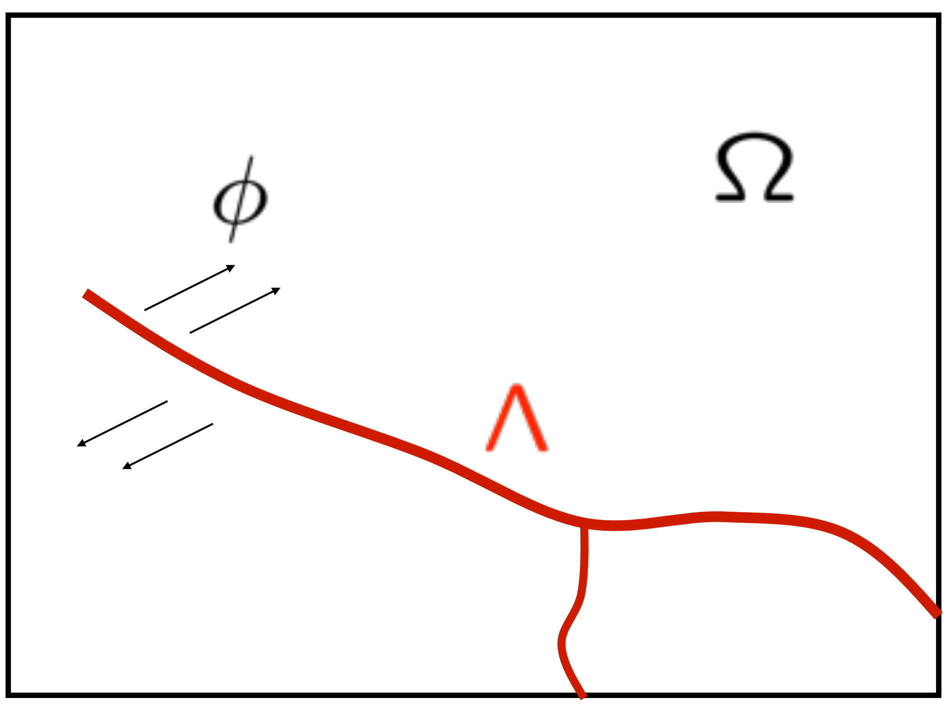
(Simulations by C.D'Angelo)

1D-3D Perfusion Model

A realistic time-dependent 1D-3D model:

$$
\begin{cases}
C_t \dfrac{\partial}{\partial t} p_t + \nabla \cdot (K_t \nabla p_t) + \alpha p_t - \phi(p_t, p_v)\delta_\Lambda = 0 \quad t > 0, \ \ \mathbf{x} \in \Omega, \\
\dfrac{\partial}{\partial t}\begin{bmatrix} p_v \\ q_v \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{c} \\ \frac{1}{l} & 0 \end{bmatrix} \dfrac{\partial}{\partial s}\begin{bmatrix} p_v \\ q_v \end{bmatrix} + \begin{bmatrix} \frac{1}{c}\phi(p_t, p_v) \\ r q_v \end{bmatrix} = \mathbf{0}, \qquad t > 0, s \in \Lambda,
\end{cases}
$$





Friday, April 5,

1D-3D Perfusion Model

A realistic time-dependent 1D-3D model:

$$\begin{cases} C_{\mathrm{t}} \dfrac{\partial}{\partial t} p_{\mathrm{t}} + \nabla \cdot (K_{\mathrm{t}} \nabla p_{\mathrm{t}}) + \alpha p_{\mathrm{t}} - \phi(p_{\mathrm{t}}, p_{\mathrm{v}}) \delta_\Lambda = 0 & t > 0, \ \mathbf{x} \in \Omega, \\ \dfrac{\partial}{\partial t} \begin{bmatrix} p_{\mathrm{v}} \\ q_{\mathrm{v}} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{c} \\ \frac{1}{l} & 0 \end{bmatrix} \dfrac{\partial}{\partial s} \begin{bmatrix} p_{\mathrm{v}} \\ q_{\mathrm{v}} \end{bmatrix} + \begin{bmatrix} \frac{1}{c} \phi(p_{\mathrm{t}}, p_{\mathrm{v}}) \\ r q_{\mathrm{v}} \end{bmatrix} = \mathbf{0}, & t > 0, s \in \Lambda, \end{cases}$$

- $p_{\mathrm{t}} : \Omega \to \mathbb{R}$ blood pressure in the tissue (3D)
- $p_{\mathrm{v}} : \Lambda \to \mathbb{R}$ blood pressure in the vessel (1D)
- $q_{\mathrm{v}} : \Lambda \to \mathbb{R}$ blood flow rate in the vessel (1D)
- $\phi : \Lambda \to \mathbb{R}$ the exchange term

1D-3D Perfusion Model

Flow model

$$\begin{cases} C_t \frac{\partial}{\partial t} p_t + \nabla \cdot (K_t \nabla p_t) + \alpha p_t - \phi(p_t, p_v)\delta_\Lambda = 0 \quad t > 0, \ \mathbf{x} \in \Omega, \\ \frac{\partial}{\partial t}\begin{bmatrix} p_v \\ q_v \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{c} \\ \frac{1}{l} & 0 \end{bmatrix} \frac{\partial}{\partial s}\begin{bmatrix} p_v \\ q_v \end{bmatrix} + \begin{bmatrix} \frac{1}{c}\phi(p_t, p_v) \\ r q_v \end{bmatrix} = \mathbf{0}, \qquad t > 0, s \in \Lambda, \end{cases}$$

- Venous transmission coefficients [range: 10^{-3} kPa^{-1} s^{-1}]
- Capillary compliance [10^{-3} kPa^{-1}]
- Tissue conductivity [0.05 mm^{2} kPa^{-1} s^{-1}]
- Vessel Windkessel parameters

1D-3D Perfusion Model

1D-3D mass transport and diffusion models:

$$\begin{cases} \dfrac{\partial}{\partial t} u_t - D_t \Delta u_t + \mathbf{v} \cdot \nabla u_t - \theta(u_t, u_v)\delta_\Lambda = f, \; t > 0, \; \mathbf{x} \in \Omega, \\ A_0 \dfrac{\partial}{\partial t} u_v - A_0 D_v \dfrac{\partial^2 u_v}{\partial s^2} + q_v \dfrac{\partial}{\partial s} u_v = 0, \qquad t > 0, s \in \Lambda, \end{cases}$$

$u_t : \Omega \longrightarrow \mathbb{R}$ mass concentration in the tissue (3D)

$u_v : \Lambda \longrightarrow \mathbb{R}$ mass concentration in the vessel (1D)

$\theta = \frac{1}{\epsilon}(u_v - \bar{u}_t)$ is a penalization term.

- Same form as ϕ
- Enforces $u_v = \bar{u}_t$