Geometric multiscaling in the circulatory system

**Local:** 3D FSI flow model

**Global:** 1D network of arteries and veins (Euler hyperbolic system)

**Global:** 0D capillary network (DAE system)
Geometric multiscaling in the circulatory system

A preview

3D Navier-Stokes (F) + 3D ElastoDynamics (V-W)

1D Euler (F) + Algebraic pressure law

0D lumped parameters (system of linear ODEs)

Acknowledgements: L. Formaggia, F. Nobile, A. Veneziani
At a bifurcation we prescribe
✓ Continuity of total pressure: $p_{t,1} = p_{t,2} = p_{t,3}$
✓ Conservation of mass: $\Sigma_i Q_i = 0$

Mathematical Analysis

The coupled problem satisfies a stability estimate similar to that of the single artery model. No shock waves developing, explicit form of characteristic variables available

In principle, it is possible to account for curvature, tapering, and for the bifurcation angle through an energy loss term.

In practice, this has a minor impact on numerical results
Peripheral branches:
Absorbing b.c
Colormap: A/A₀-1
1D model of circulation: peripheral resistance

\[
\frac{\Delta P(t)}{\Delta P} = R Q(t)
\]

Flow rate (cm^3/s)

Pressure (dyne/cm^2)

LifeV

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3D-1D for the carotid artery: pressure pulse

(Simulation by A. Moura)
A geometric multiscale approach for the circulatory system

Superior mesenteric

Right renal

Abdominal aorta D

Thoracic aorta B

Celiac A

Left renal

Superior mesenteric

Right renal

Abdominal aorta D

Thoracic aorta B

Celiac A

Left renal
Geometric Multiscale - Upper Aorta

**Models:**
- 3-D FSI Aorta
- 1-D arterial tree
  - 92 tapered elements
  - viscoelastic wall
- 0-D terminals
  - 47 Windkessel elements (RCR)

**Coupling:**
- averaged/integrated quantities at the interfaces (flow rate or normal stress)
- segregated approach for the solution of the coupled problem (Newton, inexact-Newton, or Broyden methods)
The 1D network coupled with a 3D domain

Time: 0.005 (s)

Pressure (dyn/cm²)
180,000
175,000
170,000
165,000
160,000
155,000
150,000
145,000
140,000
135,000
130,000
125,000
120,000
115,000
110,000
105,000
100,000
95,000
90,000
85,000
80,000
75,000
70,000
65,000
60,000

Displacement (cm)
0.1
0.2
0.3

Velocity (cm/s)
0
25
50
75
100

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3D-FSI/1D coupling has been using for 15 years as an **effective strategy** to simulate large portions of the cardiovascular tree


Is the solution obtained with this strategy **consistent** with the full 3D-FSI one?

Few works in this direction:

- Formaggia, Moura, Nobile, M2AN 07: cylindrical domain
- Blanco et al, J. Biomech 09: real geometry with membrane structure
Energy preserving coupling conditions

3D-FSI model energy:

\[ E^{3D}(t) = \frac{\rho_f}{2} \int_{\Omega_f} |u_f(t, x)|^2 \, d\Omega + \int_{\Omega_s} \frac{\rho_s}{2} |\tilde{\eta}_s(t, x)|^2 \, d\Omega + \]

\[ \int_{\Omega_s} W(E(t, x)) \, d\Omega + \int_{\Gamma_{out}} \alpha_e |\tilde{\eta}_s(t, x)|^2 \, d\gamma, \]

\( W \) being the elastic energy,
\( \alpha_e \) the elastic coefficient of the surrounding tissue

1D model energy:

\[ E^{1D}(t) = \frac{\rho_f}{2} \int_0^L A(t, x) U^2(t, x) \, dx + \int_0^L \Psi(A(t, x)) \, dx, \]

\[ \Psi(A) = \int_{A_0}^A \psi(\tau) \, d\tau \quad \Psi \text{ being the algebraic law} \]
Proposition. If the interface conditions are such that

$$\Delta \mathcal{E}(t) = \int_{\Gamma_f^t} T_f(u_f(t), p_{tot}(t)) n \cdot u_f(t) \, d\gamma +$$

$$\int_{\Gamma_s^t} T_s(\eta_s(t)) n \cdot \dot{\eta}_s(t) \, d\gamma + Q(t)|_{z=0} P_{tot}(t)|_{z=0} \leq 0$$

for all $t > 0$, then the coupled 3D-FSI/1D problem satisfies the energy decay property

$$\frac{d}{dt} \left( \mathcal{E}^{3D}(t) + \mathcal{E}^{1D}(t) \right) \leq 0$$

where $P_{tot} = P + \frac{\rho_f}{2} U^2 \quad p_{tot} := p + \frac{\rho_f}{2} |u_f|^2$ are the total pressures.
Energy preserving coupling conditions

**Proposition** The following interface conditions

\[
\begin{align*}
\int_{\Gamma_f} u_f(t) \cdot n \, d\gamma &= Q(t)|_{z=0}, \\
(T_f(u_f(t), p_{tot}(t))n)|_{\Gamma_f} &= -P_{tot}(t)|_{z=0}n,
\end{align*}
\]

for the fluid part, joined with either

\[
T_s(\eta_s)n = 0 \quad \text{at} \quad \Gamma_s, \quad \text{or} \quad \begin{cases} 
\eta_s \cdot n = 0 & \text{at} \quad \Gamma_s, \\
(T_s(\eta_s)n) \times n = 0 & \text{at} \quad \Gamma_s,
\end{cases}
\]

for the structure, are energy preserving.

*(L. Formaggia, A. Quarteroni, C. Vergara, On the physical consistency between three-dimensional and one-dimensional models in haemodynamics, JCP, 2012)*
How to include the effect of the surrounding tissue in the 1D model?

\[ K_{1D} = \beta = \frac{H_s E \pi}{(1 - \nu^2)A} \]
\[ K_{ST} = \alpha_e \]

\( E \) being the Young modulus, \( H_s \) the structure thickness, \( \alpha_e \) being the elastic coefficient of the surrounding tissue.

We consider **two elastic springs in parallel**, \( K_{1D} \) modeling the 1D model and \( K_{ST} \) modeling the surrounding tissue. This leads to the following **equivalent elasticity**

\[ K_{eq} = K_{1D} + K_{ST} \]

and then to the following **equivalent Young modulus** for the 1D model

\[ \hat{E} = E + \frac{(1 - \nu^2)A}{H_s \pi} \alpha_e \]
3D numerical results:

2 test cases in the cylinder coupled at the outlet with a 1D model and with the following flux inlet condition.

\[ Q = \begin{cases} \sin^2(\pi t / T) & t \leq T/2, \\ 0 & t > T/2, \end{cases} \]

By computing the mean pressure at the middle section \( S \) of the cylinder, we obtain **big spurious reflections when neglecting the ST** in the 1D model.

\[ \alpha_e = 10^6 \text{ dyne/cm}^3 \quad \text{and} \quad \alpha_e = 5 \cdot 10^6 \text{ dyne/cm}^3 \]