

Master International de Mécanique, 2018-19

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Version 1.

Abstract

Pick a project, report your choice to F.H.. to make sure that no one else has selected the same before you. Project reports are to be handed in .pdf format at the time of the exam. There will be a small bonus for those who use L^AT_EX to make their report.

List of CFD Projects using freefem++

Project 2***: Swimming of micro-organisms

Complete exercise of Stokes flow; the object swim is a Naca012 wing with a force movement of the with the following deformation $[x, y] \rightarrow [x, y + x^2 * \beta]$ with $\beta = 0.2 * \sin(t)$,

the freefem++ code to build these king meshes:

```
real beta =0.1;
real x0 = 0, y0=0, Cx =0.2, Cy=0,theta=0;           // 30*pi/180.;

func bool MoveWing(real cx,real cy,real ct,real st,real x0,real y0)
{
    real X=x-cx,Y=y-cy;                               // save x,y;
    x = X*ct + Y*st + cx + x0;
    y = -X*st + Y*ct + cy + x0;
    return 1;
}

func Naca012=0.17735*sqrt(x)-0.075597*x
- 0.212836*(x^2)+0.17363*(x^3)-0.06254*(x^4);

border C(t=0,2*pi) { x=0.5+5.*cos(t); y=5.*sin(t);label=1}
border Splus(t=0,1) { x = t; y = beta*t^2 + Naca012;
```

```

        MoveWing(Cx,Cy,cos(theta),sin(theta),x0,y0) ; label=2;}
border Sminus(t=1,0){ x = t; y= beta*t^2 - Naca012;
        MoveWing(Cx,Cy,cos(theta),sin(theta),x0,y0) ; label=2;}
plot(Splus(70)+Sminus(70),wait=1);

for(real cc = -0.3; cc<0.3; cc += 0.01)
{
    verbosity=0;
    beta = cc;
    theta = cc*100*pi/180.; // angle
    plot(Splus(70)+Sminus(70),wait=0);
}

```

Q0 Compute the force on the Objet, and the barycenter of the wing

Q1 Make a plot of velocity components and rotation angle versus time.
Choose a velocity of the two line, such that the objet move.

Q2 Make a film of the swimming

Project 3: Comparison between inviscid and viscous flow

Q1. Use Geometry 1 (ellipse in a rectangle) with $\alpha = 20^\circ$ and Equation set 2 (Navier-Stokes). Then compute the flow at Reynolds number 50.

Q2. Then on the same geometry use Equation set 1 (irrotational flow) and try to find the constant value for ψ on the ellipse that makes the flow nearest to the result of Question 1.

Q3. Compute the lift of the ellipse by using eq. set 2 (Navier-Stokes) at Reynolds number 50, 100, 200, $+\infty$. Use the lift to compute the appropriate constant for ψ on the ellipse and plot the results for these values.

Project 6*: mixing

With Geometry 2 (Y-shape tube)

Q1: Compute the flow with Equation set 1 (irrotational flow), parabolic inflow at the top of both branches of the Y, no slip condition on the sides and free outflow at the bottom.

Q2: Assume that on one branch the liquid entering the pipe is blue and on the other branch red. Using an equation for the transport of a chemical compute and display the mixing.

Q3: Same with Stokes flow

Q4: Compare the FEM-Characteristics with another numerical scheme for the transport equation (you may if you wish add a small diffusion term).

Project 8: Buoyancy**

(Here you need a better computer. Those of Atrium are not sufficient.)

The geometry is the unit square with Equation set 2 (Navier-Stokes) coupled with the temperature

$$\partial_t \theta + u \nabla \theta - \nabla \cdot (\kappa \nabla \theta) = \frac{\nu}{2} |\nabla u + \nabla u^T|^2.$$

With the Boussinesq approximation a force f is added to the right hand side of Equation set 2 (Navier-Stokes): $f_2 = -C\theta$.

Q1: assuming free slip for the velocity at the 4 walls of the cavity, and room temperature everywhere except at the vertical wall where it is 100° on right and 0° on the left, and compute the flow.

Q2: Same with no slip conditions at the wall

Q3: Change your numerical scheme for the temperature equation and redo the calculations, with full implicit schema and using Newton methods to solve the no-linearity problem.

Project 9***: Richtmayer instabilities

The domain is the unit square, the initial state is the heavy fluid on top of the light fluid with an interface of equation $y = \sin(ax)$.

Q1. Let the flow evolve in time and show the results.

Q2 Same but with mesh adapted to ρ .

Q3. Study the influence of a .

Project 10***: Kelvin-Helmholtz instabilities

The geometry is a rectangle $(0, L) \times (-H, H)$. The inflow conditions are $u \cdot n = -1$ when $y > 0$ and zero otherwise. There is free slip on the lateral walls and free outflow. The fluid velocity at time zero is $u_2 = 0$, $u_1 = 0$ if $y < 0$ $u_1 = 1$ otherwise

Q1: Compute the flow; if it is stationary perturb it and see how the instabilities grow.

Q2: Use a stream function - vorticity formulation to compute the same.

Q3: Redo Q1 till time T then continue but with periodic conditions between the inflow and the outflow.

Project 13**: Euler versus Navier-Stokes

Geometry 1 (ellipse in a rectangle) but with the ellipse replaced by a wing profile NACA0012 (equation is given in example 3.9 of freefem++ documentation).

Q1: compute the lift by using Equation set 1 (irrotational flow).

Q2 : Compute the flow using Equation set 2 (Navier-Stokes)

Q3: Set $\nu = 0$ in Q2 and see if you can find the same solution as in Q1?

Project 14:** A pseudo rotating geometry

The geometry is an ellipse centered at the origin of radius a, b and the long axis is horizontal; the fluid is bounded by a circle centered at the center of the ellipse and of radius $R \gg a$. Use equation set 2 (Navier-Stokes). with boundary condition on the large circle $u = (\cos\alpha, \sin\alpha)^T$.

Q1: Compute the flow and the resulting force df on the ellipsoid elementary area dS .

Q2: Change α by writing Newton's law on the ellipse discretized by an explicit time scheme.

Q3: Check whether the fact that we are using a frame of reference attached to the ellipse requires a Coriolis term added to the Navier-Stokes equation and do the time dependent calculation of the flow in the frame of reference where the ellipse is fixed but with visualization in the frame of reference where the ellipse rotates,

Project 15*:** A rotating Geometry

With Geometry 1 (ellipse in a rectangle) we will let the ellipse E rotate freely around its center. It is the same project as Project 11 but we do not change the reference frame. Since this project is harder than 11 you may team with 11.

Q1: Compute the flow using Equation set 2 (Navier-Stokes) and compute the resulting force df on the ellipsoid elementary area dS .

Q2: Change α by writing Newton's law on the ellipse discretized by an explicit time scheme.

Q3: Include in a time loop Q1,Q2 and a remeshing step to allow for the geometry change.

Project 16: A compressible flow

Use Geometry 3 (nozzle with a bump) with slip conditions on the top and bottom boundaries, uniform inflow u_∞ normal velocity and free outflow.

Q1: Compute the flow using Equation set 1 (irrotational flow).

Q2: Same but with a potential function ϕ instead of ψ (equation set 0).

Q3: Use the transonic equation $\nabla \cdot (1 - |\nabla\phi|^2)^\beta \nabla\phi = 0$ (see chapter 2 in the notes) and compute the flow by a fixed point method:

$$\text{Loop on m: } \nabla \cdot (1 - |\nabla\phi^m|^2)^\beta \nabla\phi^{m+1} = 0$$

Start with u_∞ small and enlarge it till a shock appears.

Project 17*: Numerical Methods

Geometry 1 (ellipse in a rectangle):

Q1: Solve the Navier-Stokes equation with P2/P1

Q2: Compare with P1b/P1 and also with the projection scheme of NSproject.edp

Q3: Don't use characteristic-Galerkin but use Newton on the time independent problem

Project 19*: Flow Through Porous Media

The geometry is a rectangle $(0, L) \times (0, H)$. The equations are

$$\nabla(\mathbf{K}\nabla p) = 0, \quad u = K\nabla p, \quad \partial_t c + u \cdot \nabla c - \nu \nabla(K\nabla c) = f \quad (1)$$

where p is the hydrostatic pressure, c the concentration of the pollutant, u the velocity of the flow, \mathbf{K} the permeability tensor of the medium, ν the diffusivity of the pollutant and f the pollutant source. The medium is an horizontal layer of clay in $(0, L) \times (0, H/3)$ with an horizontal layer of limestone over it. First we suppose that \mathbf{K} is the identity matrix times a scalar K which in the clay is 1000 and K in the limestone is 1; $\nu = 1.0e - 3$. p on the boundary is $x + y$, which $\partial c / \partial n$ is imposed on the boundary at all times. The source term is a Dirac at $x = L/2, y = H/4$, constant in time.

Q1: Use P1 elements for both u and c and compute the evolution of c . Compute the integral of c in the domain and plot it as a function of time.

Q2: Use a discontinuous Galerkin method to improve conservativity

Q3: Now \mathbf{K} is a diagonal matrix with $K_{11} = 10K$ and $K_{22} = K$. Redo Q1 and Q2.

Project 21**: Free Boundary

The problem is to find the Free Boundary of the river when a wake enter on the river.

To simplify the coordinate is x, y (y corresponding to the z axis with gravity).

the domain is a kind of rectangle with up free surface.

The equation are Stokes or Navier-Stokes, the 3 left, down, right boundary condition are $u = 0$ and on the top, $u = \text{given}$ and $\sigma.n = \text{given}$.

Use the free surface equation equation of FreeFem++ as a good starting point.

Project 22: Turbulence**

The geometry is a backward step: $(0, a) \times (c, d) \cup (a, b) \times (0, d)$. The flow is modeled by the set 3 (RANS equations). Compute a turbulent flow with uniform inflow conditions and free outflow conditions.

Q1: Begin with Navier-Stokes

Q2: Then add the equation for k .

Q3 Find on the web or in the literature a similar test case (but not the same turbulence model) and compare.

Project 23: Generation of a axisymmetric droplet very viscous

The problem is a times depend Stokes in moving domain in green defined by a level set function. You have forces at the top to push the fluid , no sleep boundary condition on boundary mold, on free surface just surface tension proportional to the radius of curvature.

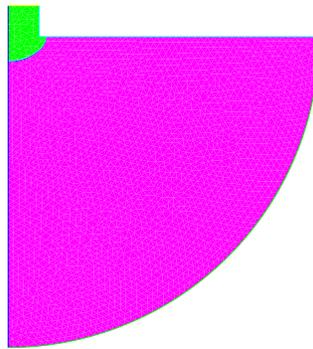


Figure 1: Domain Ω et Ω_t

Definitions

Geometry 1 (ellipse in a rectangle) An ellipse in a long rectangle $(0, L) \times (0, H)$. The ellipse is centered at $(L/5, H/2)$ and has a long radius a and a short radius b and the long axis makes an angle α with the horizontal axis of the reference frame.

Geometry 2 (Y-shape tube) A bifurcating tube shaped like the letter Y. The parameters are the lengths a and b of the 3 portions of the tubes, the diameter d and the angle α between the 2 branches of the Y.

Geometry 3 (nozzle with a bump) A long rectangle $(0, L) \times (0, H)$ where the flat bottom boundary has a bump shaped as a portion of circle, $x(t) = \frac{L}{3} + R \cos(\beta t)$, $y(t) = -R + a + R \sin(\beta t)$, $t \in (t_1, t_2)$; naturally t_1, t_2 needs to be calculated as functions of R, β so that the boundary is closed and simply connected.

Equation set 0: potential flow The flow is assumed irrotational and inviscid so that $-\Delta\phi = 0$.

The flow is $u = \nabla\phi$, $p = p_0 - \frac{1}{2}|u|^2$; it is parallel to the walls so $\partial_n\phi = 0$ on the boundaries where there is a slip condition $u \cdot n = 0$. Typical inflow/outflow condition is ϕ constant.

Equation set 1 (irrotational flow) The velocity of the flow and the pressure are $u = \nabla \times \psi$, $p = p_0 - \frac{1}{2}|u|^2$. The flow is assumed irrotational and inviscid so that $-\Delta\psi = 0$.

The flow is parallel to the walls so ψ is constant on the boundaries where there is a slip condition $u \cdot n = 0$. The inflow is assumed parabolic and compatible with the slip conditions. The outflow condition is $\partial_n\psi = 0$.

Equation set 2 (Navier-Stokes): Navier-Stokes flow Navier-Stokes flow with slip conditions on the walls (i.e., $u \cdot n = 0$, $\partial_n(u \cdot s) = 0$, s being the tangent, n the normal), parabolic inflow and free outflow (i.e. $-\nu\partial_n u + pn = 0$).

Equation set 3: RANS model A one equation RANS turbulence model to compute the turbulent viscosity in the Reynolds averaged equations by $\nu_T = \nu + c_1\sqrt{k}$ where k is the turbulent kinetic energy :

$$\partial_t u + u \nabla u - \nabla \cdot (\nu_T (\nabla u + (\nabla u)^T)) + \nabla p = 0, \quad \nabla \cdot u = 0$$

$$\partial_t k + u \nabla k - c_1 \nabla(\sqrt{k} \nabla k) = c_1 \sqrt{k} \frac{1}{2} |\nabla u + \nabla u^T|^2 \quad (2)$$

with $\partial_n k = 0$ on walls and at the outflow boundaries and initial and inflow conditions given. For the velocity no slip conditions on the walls and parabolic inflow and free outflow (i.e. $-\nu_T \partial_n u + pn = 0$).

Equation set 5: piecewise incompressible viscous flow Incompressible Navier-Stokes flow with multiple regions of different densities is modeled by

$$\begin{aligned} \partial_t(\rho u) + \nabla \cdot (\rho u \otimes u) + \nabla p - \mu \Delta u &= 0 \\ \partial_t \rho + \nabla \cdot (\rho u) &= 0, \quad \nabla \cdot u = 0 \end{aligned} \quad (3)$$

ρ constant in each region