Numerical Modelling of the Air Flow in the Respiratory Tract

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Outline

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   - The Distal Part
   - The Spring
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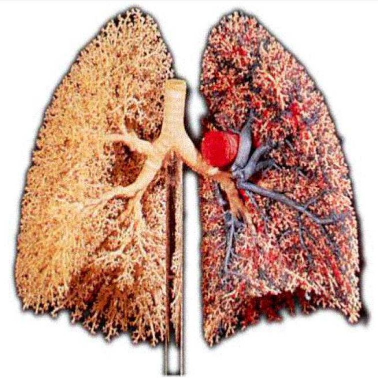
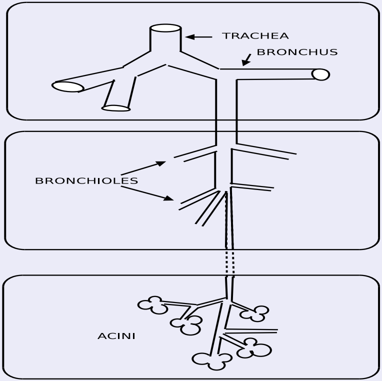
   - 2D Simulation Results: FreeFEM++2d
   - 3D Simulation Results: FreeFEM++3d

Outline

Modelling the Air Flow in the Lung

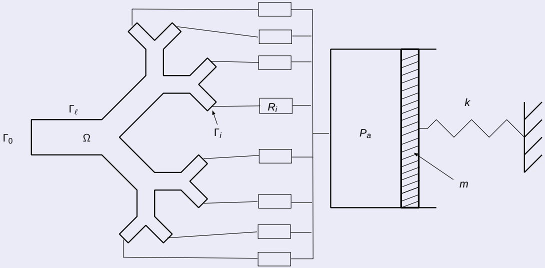




- Decomposition of the Respiratory Tree in **3 Parts**

A Three-Level Model 1/2

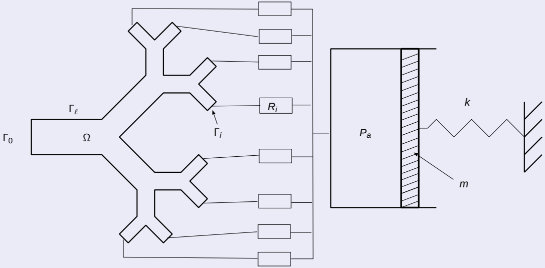
Multiscale Model



- Model introduced by C. Grandmont, Y. Maday and B. Maury '05
- blood flow :
  - C. A. Figueroa, K.E. Jansen, C. A. Taylor, I. E. Vignon-Clementel '06
  - A. Quarteroni, S. Ragni, A. Veneziani '01
  - A. Quarteroni, A. Veneziani '03

A Three-Level Model 2/2

Multiscale Model



- The proximal part (up to the 5-7th generation) : where the incompressible Navier-Stokes equations hold
- The distal part (from the 6-8th to the 16 generation) : where the Poiseuille law is satisfied
- The acini : where the oxygen diffusion takes place, embedded in an elastic medium : the parenchyma described by a simple spring model.

The Proximal Part

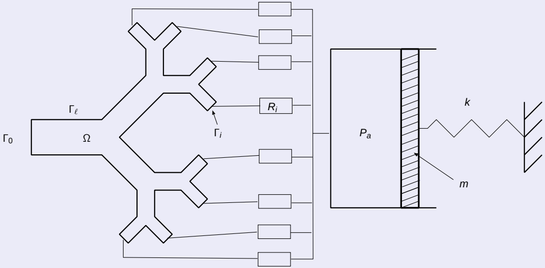
Navier-Stokes Equations

$$\left\{\begin{array}{rcll} \rho\dfrac{\partial u}{\partial t} + \rho(u\cdot\nabla)u - \mu\Delta u + \nabla p &=& 0, & \text{in } \Omega, \\ \nabla\cdot u &=& 0, & \text{in } \Omega, \\ u &=& 0, & \text{on } \Gamma_\ell, \\ \mu\nabla u\cdot n - pn &=& 0, & \text{on } \Gamma_0, \quad \text{(nose)} \\ \mu\nabla u\cdot n - pn &=& -\Pi_i n, & \text{on } \Gamma_i, \quad \mathrm{i} = 1,\ldots,\mathrm{N}. \end{array}\right.$$

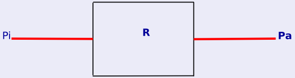
The pressures Π_i are unknown and depend on the downstream parts.

The Distal Part

Multiscale model



Electric Circuit Analogy

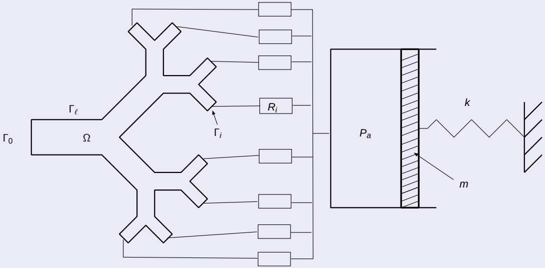


Poiseuille Law

$$\Pi_i - P_a = R_i \int_{\Gamma_i} u \cdot n$$

The Spring

Multiscale model



Spring-mass

The equation satisfied by the position x of the diaphragm is

$$m\ddot{x} = -kx + f_{mus} + P_a S$$

The Coupled System

Navier-Stokes / Spring-Mass

$$
\left\{
\begin{array}{rcll}
\rho\dfrac{\partial u}{\partial t} + \rho(u\cdot\nabla)u - \mu\Delta u + \nabla p & = & 0, & \text{in } \Omega, \\
\nabla\cdot u & = & 0, & \text{in } \Omega, \\
u & = & 0, & \text{on } \Gamma_\ell, \\
\mu\nabla u\cdot n - pn & = & 0, & \text{on } \Gamma_0, \\
\mu\nabla u\cdot n - pn & = & -P_a\, n - \left(R_i \displaystyle\int_{\Gamma_i} u\cdot n\right) n, & \text{on } \Gamma_i, \\
m\ddot{x} & = & -kx + f_{mus} + SP_a, & \\
\text{By incompressibility} \quad S\dot{x} & = & \displaystyle\sum_{i=1}^{2^N}\int_{\Gamma_i} u\cdot n = -\int_{\Gamma_0} u\cdot n &
\end{array}
\right.
$$

NB : One particularity of this system that all of the outlets Γ_i are **coupled**

Outline

Motivation

Bilinear Form

The special boundary conditions modify the standard Stokes-bilinear forms :

$$a_m(u,v) = \mu \int_\Omega \nabla u \cdot \nabla v + \sum_{i=1}^{N} R_i \left(\int_{\Gamma_i} u \cdot n \right) \left(\int_{\Gamma_i} v \cdot n \right)$$

Finite Element Discretization $\Longrightarrow$

- All the elements of each boundary Γ_i are **coupled**
- The FE matrix obtained is **non standard**

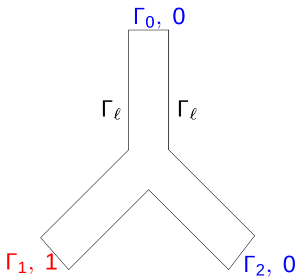
This cannot be easily and directly implemented in a standard FEM Software without going deeply into the code : L. Baffico, C. Grandmont, B . Maury '09

Superposition Method.

Same idea used by :

- J. Heywood, R. Rannacher, S. Turek '96
- L. Formaggia, J.-F. Gerbeau, F. Nobile, A. Quarteroni '02
- A. Veneziani, C. Vergara '07

Numerical Method 1/3



Pre-Processing Step

We compute the solutions (u_i, p_i) of the following generalized Stokes Problem

$$\left\{\begin{array}{rcll} \frac{\rho}{\delta t} u_i - \mu \Delta u_i + \nabla p_i & = & 0, & \text{in } \Omega, \\ \nabla \cdot u_i & = & 0, & \text{in } \Omega, \\ u_i & = & 0, & \text{on } \Gamma_\ell, \\ (\mu \nabla u_i - p_i) n & = & -n, & \text{on } \Gamma_i, \\ (\mu \nabla u_i - p_i) n & = & 0, & \text{on } \Gamma_j,\ j \neq i. \end{array}\right.$$

Prediction Step

The correction term $(\tilde{u}^{n+1}, \tilde{p}^{n+1})$ takes into account the unsteady term and the time dependent spring term :

$$\left\{ \begin{array}{rcll} \frac{\rho}{\delta t}\tilde{u}^{n+1} - \mu\Delta\tilde{u}^{n+1} + \nabla\tilde{p}^{n+1} & = & \frac{\rho}{\delta t}u^n \circ X^n, & \text{in } \Omega, \\ \nabla \cdot \tilde{u}^{n+1} & = & 0, & \text{in } \Omega, \\ \tilde{u}^{n+1} & = & 0, & \text{on } \Gamma_\ell, \\ (\mu\nabla\tilde{u}^{n+1} - \tilde{p}^{n+1})n & = & 0, & \text{on } \Gamma_i, \; i = 0, \ldots, N. \end{array} \right.$$

Where $u^n \circ X^n = u^n(x - u^n(x)\delta t)$

Numerical method 3/3

Correction Step

Thanks to the linearity of the problem, the solution at the time step $n+1$ is computed as follows :

$$u^{n+1} = \tilde{u}^{n+1} + \sum_{i=0}^{2^N} \alpha_i^{n+1} u_i$$

where $\alpha^{n+1} = \left(\alpha_1^{n+1}, \ldots, \alpha_N^{n+1}\right)$ solves a linear system : $A\alpha = b$

Note that α^{n+1} is such that the boundary conditions on Γ_i

$$\mu \nabla u \cdot n - pn = -P_a n - \left(R_i \int_{\Gamma_i} u \cdot n\right) n$$

are satisfied on the Γ_i.

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Numerical method 3/3

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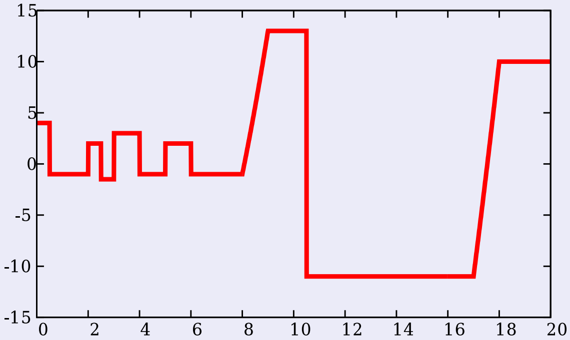
Outline

2D Simulation of the Respiration

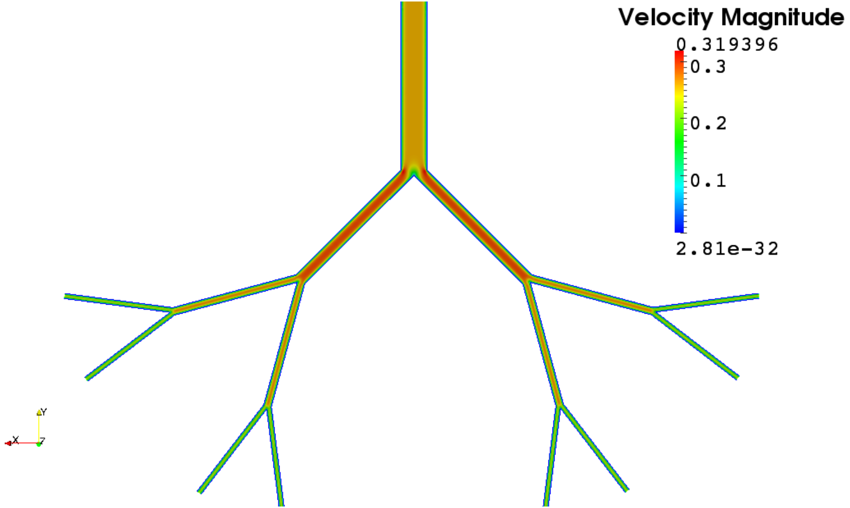
Forced Respiration

We present numerical results obtained in the case of forced maneuvers.
In this case the force f_{mus} applied to the spring is as follows :

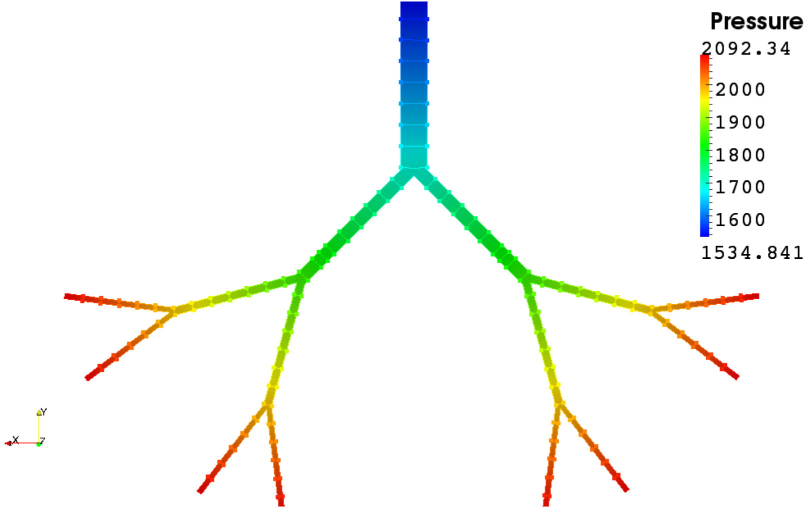


Note that the first part of the maneuver (for $0 < t < 8s$) corresponds to respiration at rest.

2D Simulation of the Respiration : Velocity

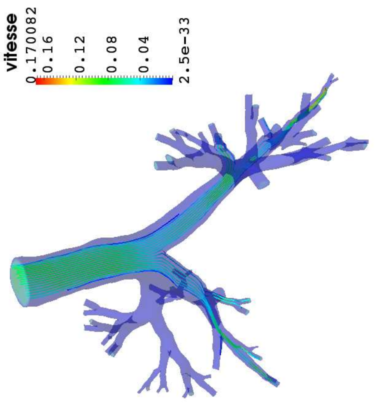


2D Simulation of the Respiration : Pressure

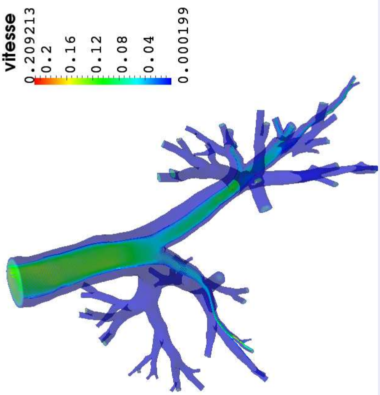


3D Simulation of the Respiration : Velocity

Velocity at inspiration

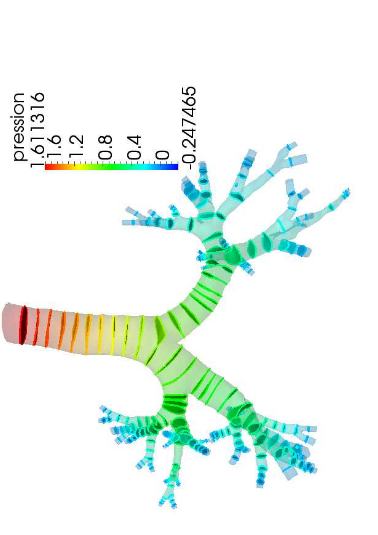


Velocity at expiration

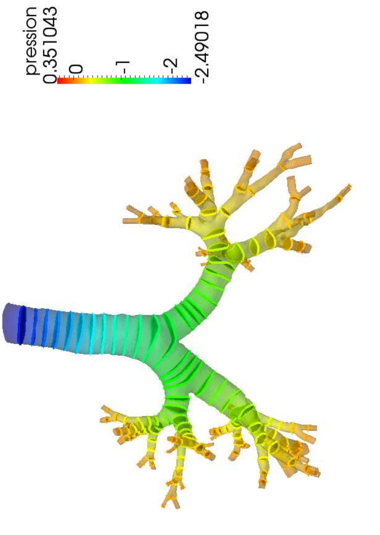


3D Simulation of the Respiration : Pressure

Pressure at inspiration

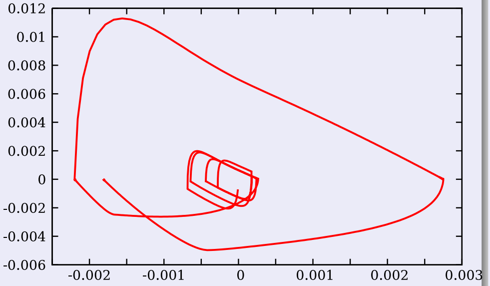


Pressure at expiration



Simulation of the Respiration: Phase Portrait

Phase Portrait: 2D



Phase Portrait : 3D