DOC-COURS IMUS RESEARCH UNIT 2

EXERCICES

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1 FreeFem++ test

(See exercice 2 of T. Chacón)

The problem is solve on the lac leman, a modified stokes problem. This problem take into account of wind on the upper part the boundary Γ_s , and the Coriolis force. and neglect all flow du to the all incoming rivers or out-coming rivers.

So the formulation is find the velocity field $\boldsymbol{u}$ and the pressure field p on domain Ω of $\mathbb{R}^3$ with the boundary Γ split in two part : Γ_s for the upper part, and Γ_b for the other part,

such that :

$$\begin{aligned}
-\nu\Delta\boldsymbol{u} + \nabla p + u\times\boldsymbol{\omega} &= 0 && \text{in } \Omega\\
\nabla\cdot\boldsymbol{u} &= 0 && \text{in } \Omega\\
\boldsymbol{u} &= 0 && \text{on } \Gamma_b\\
\boldsymbol{u}.\vec{\boldsymbol{n}} &= 0 && \text{on } \Gamma_s\\
\forall\boldsymbol{\tau}\in\vec{\boldsymbol{n}}^\perp: \quad (\nu\frac{\partial\boldsymbol{u}}{\partial\vec{\boldsymbol{n}}} + p\vec{\boldsymbol{n}}).\boldsymbol{\tau} \quad = \quad (\nu\frac{\partial\boldsymbol{u}}{\partial\vec{\boldsymbol{n}}}).\boldsymbol{\tau} &= \boldsymbol{\tau}.\boldsymbol{w}, && \text{on } \Gamma_s
\end{aligned}$$

where $\vec{\boldsymbol{n}}$ is a external unit normal, $\vec{\boldsymbol{n}}^\perp$ the orthogonal to $\vec{\boldsymbol{n}}$ sub-space of $\mathbb{R}^3$ (here $\vec{\boldsymbol{n}}^\perp = \mathbb{R}^2$).

For a simplicity we take physical parameter $\nu, \boldsymbol{\omega}, \boldsymbol{w}$ just to see the influence,

$$\nu = 0.1, \quad \boldsymbol{\omega} = \begin{vmatrix} 0\\ 0\\ \omega_z \end{vmatrix}, \quad \boldsymbol{w} = w \begin{vmatrix} cos(\theta)\\ sin(\theta)\\ 0 \end{vmatrix}$$

where $\omega_z = 0.5$ or $\omega_z = 0.$, and $w = 1$, $\theta = \pi/4$.

Q1) Write the variational formulation of the problem.

Q2) For graphic reason, we rescale the domain in z direction of a factor r_z, rewrite the equation in the rescale domain.

Q3) Write freefem++ script to solve the modified stokes equation. You will use the classical Taylor-Hood Finite Element. The 3d rescale mesh is store in file `Leman-3d.mesh` and the 2d mesh of the lac is store in file `Leman.msh`. The boundary label on this 3d mesh are 5 for Γ_s and $1, 2, 3, 4$ for for Γ_b, where this four labels are :1 for the rhone input, 2 for the rhone output, 3 for the vertical border, and 4 for the bottom part.

To compute the rescale factor r_z ,we know : the mesh is km and the true deep of the lac is $500m$.

Q4) Make a rapport with a plot of the 2 solution for $\omega_z = 0.5$ and $\omega_z = 0.0$ on plan $z = 0, z = -1, z = -2, z = 3, z = 4$ of the velocity and the pressure.

Q5) Analyses the influence of Coriolis force on this problem.