PERGAMON

Automatica 36 (2000) 587–593

www.elsevier.com/locate/automatica

Brief Paper

Exponential stabilization of an overhead crane with flexible cable via a back-stepping approach☆

B. d'Andréa-Novel[a,*], J.M. Coron[b]

[a]Centre de Robotique, École des Mines de Paris, 60 bvd Saint-Michel, 75272 Paris Cedex 06, France
[b]Université de Paris-Sud, Analyse Numérique et EDP, Bâtiment 425, 91405 Orsay Cedex, France

Received 3 February 1998; revised 27 July 1999; received in final form 13 August 1999

Abstract

This paper deals with the uniform exponential stabilization of a hybrid PDE–ODE system which describes an overhead crane with flexible cable. A previous linear boundary feedback law (see d'Andréa-Novel, Boustany, Conrad & Rao (1994). *MCSS Journal*, *1*, 1–22) depending on the platform position and velocity and on the angular displacement of the cable at the connection point to the platform, led to asymptotic stabilization but could not provide an exponential decay (see Rao (1993). *European Journal of Applied Mathematics*, *4*, 303–319). Taking advantage of the "cascaded" structure of the hybrid system, we propose here a back-stepping approach leading to a linear boundary feedback which "naturally" depends in addition, on the angular velocity of the cable. We prove that this boundary feedback law produces uniform exponential stability and illustrative simulations are displayed. In d'Andréa-Novel & Coron ((1997). Proceedings of the IFAC SYROCO '97 Conference, Nantes) this result has been established under a small gain condition on the feedback stabilizing the subsystem made of the PDE. Here, by using a result of Datko (see Datko (1970). *Journal of Mathematical Analysis and Application*, *32*, 610–616) we show that this condition can be relaxed.

Keywords: Back-stepping; Boundary control; Cascaded systems; Exponential stabilization; Flexible robots; Lyapunov methods

1. Introduction

For finite-dimensional systems, the existence of a "cascaded structure", namely chains of integrators, (see e.g. Byrnes & Isidori, 1989; Tsinias, 1989; Coron & Praly, 1991; Kanellakopoulos, Kokotovic & Morse 1992; Krstić, Kanellakopoulos & Kokotovic 1995, for this notion), is a powerful tool for the design of stabilizing controllers. What we want to emphasize is that the cascaded structure of some flexible mechanical systems coupling ODE and PDE is also a useful property in regard to stabilization. For example in Coron and d'Andréa-Novel (1998), we have proposed a class of nonlinear asymptotically stabilizing boundary feedback laws for a rotating body-beam without natural damping. In this present paper, we also use its cascaded structure to stabilize the following system, called "overhead crane" and made of a motorized platform of mass M moving along an horizontal bench. A flexible cable of length L is attached to the platform and holds a load mass m (see Fig. 1).

We make the following assumptions:

(i) The cable is completely flexible and nonstretching.
(ii) Transversal and angular displacements are small.
(iii) The acceleration of the load mass is negligible with respect to the gravitational acceleration g.

Therefore, if s denotes the arc length along the cable, $y(s,t)$ the horizontal displacement at time t of the point whose curvilinear abscissa is s, X_p the platform abscissa, $y_s(s,t)$ the angular inclination of the cable at s with respect to the vertical, ρ the mass per unit length of the cable and u the force applied to the platform, the dynamical equations of the platform and the cable take the

☆The original version of this paper was presented at the IFAC Conference SYROCO which was held in Nantes, France during 1997. This paper was recommended for publication in revised form by Associate Editor H. Nijmeijer under the direction of Editor T. Basar. Supported by a grant from DRET No. 951170 and by the PRC-GDR "Automatique" of the CNRS and the MST.

* Corresponding author. Tel.: 01-40-51-90-94; fax: 01-43-26-10-51.
E-mail address: andrea@caor.ensmp.fr (B. d'Andrea-Novel)

PII: S0005-1098(99)00182-X

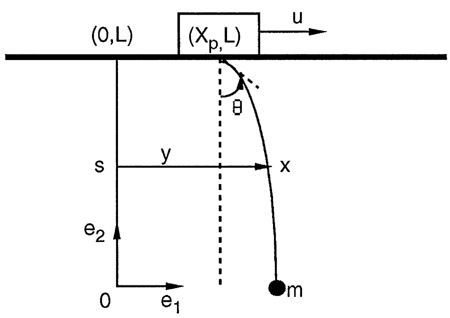

Fig. 1. The overhead crane with flexible cable.

following form (for details we can refer e.g. to d'Andréa-Novel et al., 1994):

$$y_{tt} - (ay_s)_s = 0,$$
$$y_s(0,t) = 0, \tag{1}$$
$$y(L,t) = X_p(t),$$
$$\ddot{X}_p = \lambda(ay_s)(L,t) + u/M = U, \tag{2}$$

with

$$a(s) = g\,s + \frac{gm}{\rho} \tag{3}$$

and

$$\lambda = \frac{(m + \rho L)g}{Ma(L)}. \tag{4}$$

Due to the last assumption (iii), the dynamical equation of the load mass can be approximated by

$$y_s(0,t) = 0.$$

By considering the "natural" energy of the hybrid system and a suitable class of boundary feedback laws depending on X_p, $\dot{X}_p$ and $y_s(L,t)$, a previous asymptotic stabilization result has been obtained in d'Andréa-Novel et al. (1994) for system (1)–(2) around the equilibrium solution $X_p(t) = \dot{X}_p(t) = 0$, $y(s,t) = y_t(s,t) = 0$. Unfortunately, for these linear boundary feedback laws, using an argument based on a result of compact perturbation due to Russell (1975), Rao (1993) proved that exponential stabilization could not be achieved. Nevertheless, when ignoring the ODE (2), which amounts to considering the velocity of the platform as the control variable for the subsystem made of the PDE, exponential stabilization has been obtained for the PDE system (see d'Andréa-Novel et al., 1994).

In fact, the complete PDE–ODE system (1)–(2) has a "cascaded structure". Taking advantage of this structural property, we construct in this paper a class of linear boundary feedback laws, depending on X_p, $\dot{X}_p$, $y_s(L,t)$ and *in addition* on $y_{st}(L,t)$, which produce exponential decay for a suitable "energy function". The existence of this "noncompact" term $y_{st}(L,t)$ in the feedback law precisely allows exponential stabilization. A similar conclusion has been obtained in Morgül, Rao and Conrad (1994) for a hybrid system with a Dirichlet boundary condition and in the case of the wave equation, i.e. when taking $a(s) = 1$ in Eq. (1). Let us also mention the recent work of Mifdal (1997), where the author also takes into account the load mass dynamics.

In fact, the use of stronger feedback terms is often necessary to obtain uniform stabilization, as explained e.g. in Morgül et al. (1994) for hybrid SCOLE models. But, contrarily to Morgül et al. (1994) or Mifdal (1997) where stronger terms were a priori prescribed, what we show here, is that the "cascaded structure" of our system *naturally* leads to this stronger term $y_{st}(L,t)$, when applying a back-stepping technique.

In Section 2 we briefly recall the exponential stabilization result of d'Andréa-Novel et al. (1994) concerning the subsystem made of the only PDE. Then we apply a back-stepping approach in Section 3 to obtain asymptotic stabilization for the hybrid PDE–ODE system (1)–(2). Section 4 is devoted to the main uniform exponential stabilization result for the hybrid system. Finally, we present some illustrative simulation results in Section 5 and we conclude.

2. Exponential stabilization of the PDE subsystem

We now briefly recall the exponential stabilization result of d'Andréa-Novel et al. (1994) where we ignore the dynamic's of the platform given by ODE (2). Therefore, we take the platform velocity $\dot{X}_p$ as the control variable v for the subsystem given by

$$\begin{aligned} &y_{tt} - (ay_s)_s = 0,\\ &y_s(0,t) = 0,\\ &y(L,t) = X_p(t),\\ &\dot{X}_p = v, \end{aligned} \tag{5}$$

with a still given by (3).

By considering the energy function

$$E = \frac{1}{2}\int_0^L (y_t^2 + ay_s^2)\,\mathrm{d}s + \frac{k}{2}X_p^2, \quad k > 0, \tag{6}$$

we obtain the following time derivative of E:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = y_t(L,t)[ay_s(L,t) + ky(L,t)]. \tag{7}$$

It is then easy to see that the following boundary feedback laws, expressed in terms of boundary condition at $s = L$

$$ay_s(L,t) + ky(L,t) = -f(y_t(L,t)), \tag{8}$$

where f is nondecreasing (continuous) and $f(0)=0$, are dissipative.

Namely, the closed-loop system is of the form

$$y_{tt}-(ay_s)_s=0,\quad y_s(0,t)=0,\quad (ay_s)(L,t)+ky(L,t)=-f(y_t(L,t)) \tag{9}$$

and by taking for example the following linear boundary feedback law (corresponding to $f=KId$ in (8)):

$$\bar{v}=-((ay_s)(L,t)+ky(L,t))/K,\quad K>0, \tag{10}$$

we have

$$\frac{\mathrm{d}E}{\mathrm{d}t}=-Ky_t^2(L,t)\leq 0. \tag{11}$$

Moreover, we know that the energy $E(t)$ uniformly exponentially decays to zero (see Theorem 4(i) in d'Andréa-Novel et al., 1994), which means the existence of constants $C>0$, $\mu>0$ such that, for any solution of (9)–(10)

$$E(t)\leq CE(0)\mathrm{e}^{-\mu t}. \tag{12}$$

Remark 1. Let us notice that $\bar{v}$ depends on $X_\mathrm{p}=y(L,t)$ and on the angular displacement θ of the cable at the connection point to the platform, since we have

$$\theta=-y_s(L,t). \tag{13}$$

Remark 2. To steer the platform at any nonzero set point $X_{\mathrm{p_c}}$, it is sufficient to replace $(k/2)X_\mathrm{p}^2$ in (6) by $(k/2)(X_\mathrm{p}-X_{\mathrm{p_c}})^2$.

3. Asymptotic stabilization of the hybrid system via a back-stepping approach

We first derive a stabilizing boundary feedback law for system (1)–(2) by using its cascaded structure. Then we prove the well posedness of the closed-loop system, and finally we prove the asymptotic stability. The exponential stability will be established in the following section.

3.1. Derivation of a stabilizing boundary feedback law

We consider the following Lyapunov function candidate:

$$V=E+\tfrac{1}{2}(\dot{X}_\mathrm{p}-\bar{v})^2. \tag{14}$$

By using the back-stepping approach (see e.g. Byrnes & Isidori, 1989; Tsinias, 1989; Coron & Praly, 1991; Kanellakopoulos et al., 1992; Krstić et al., 1995), the Lyapunov function candidate for the complete hybrid system is the sum of a Lyapunov function associated to subsystem (5) and of the term $\frac{1}{2}(\dot{X}_\mathrm{p}-\bar{v})^2$, where $\bar{v}$ is the feedback law given by (10), ensuring the stabilization of subsystem (5).

Directly computing the back-stepping control law leads to

$$U=\dot{\bar{v}}-\alpha(\dot{X}_\mathrm{p}-\bar{v})-(ay_s)(L,t)-ky(L,t),\quad \alpha>0. \tag{15}$$

This boundary feedback law ensures asymptotic stability, but it is not clear if it produces exponential stability. More precisely, Lemma 4 given below, does not hold with this feedback (15).

Inspired by Morin and Samson (1997), we propose the following boundary feedback law U, which will exponentially stabilize the hybrid system, for $\alpha>0$ sufficiently high (see Theorem 3)

$$U=\dot{\bar{v}}-\alpha(\dot{X}_\mathrm{p}-\bar{v}),\quad \alpha>0, \tag{16}$$

which gives, from (2)

$$u=M(U-\lambda(ay_s)(L,t)). \tag{17}$$

More precisely we establish the following lemma.

Lemma 1. *The linear boundary feedback control u given by* (17) *where U is given by* (16) *and* $\bar{v}$ *by* (10) *makes* $V(t)$ *decrease for* $\alpha>K/2$.

Proof. Computing the time-derivative of V gives

$$\dot{V}=-\alpha(\dot{X}_\mathrm{p}-\bar{v})^2-((ay_s)(L,t)+kX_\mathrm{p})^2/K+(\dot{X}_\mathrm{p}-\bar{v})((ay_s)(L,t)+kX_\mathrm{p}), \tag{18}$$

consequently, we have

$$\dot{V}\leq-\frac{((ay_s)(L,t)+kX_\mathrm{p})^2}{2K}-\left(\alpha-\frac{K}{2}\right)(\dot{X}_\mathrm{p}-\bar{v})^2 \tag{19}$$

and $\dot{V}\leq 0$ provided that $\alpha>K/2$, which ends the proof of the lemma. □

This is a formal computation. We must now check that the hybrid system (1)–(2) in closed-loop form with (17), (16), (10) corresponds to a well-posed problem, associated with a maximal monotone operator.

Remark 3. In the expression of the feedback law u, $\dot{\bar{v}}$ gives rise to the term $\dot{\theta}$. As already mentioned in the introduction, the existence of this noncompact term $y_{st}(L,t)$ will allow one to conclude to exponential stabilization.

3.2. Well posedness

We set the PDE–ODE system (1)–(2) in closed-loop form with (17), (16), (10) into the state-space form

$$\dot{X}+AX=0, \tag{20}$$

with

$$X = \begin{pmatrix} y = y(s,t) \\ z = y_t(s,t) \\ b = y(L,t) = X_\mathrm{p} \\ \eta = \dot{X}_\mathrm{p} - \bar{v} = z(L,t) + \dfrac{(ay_s)(L,t) + kb}{K} \end{pmatrix}, \tag{21}$$

and

$$AX = \begin{pmatrix} -z \\ -(ay_s)_s \\ -\eta + ((ay_s)(L,t) + kb)/K \\ \alpha\eta \end{pmatrix}. \tag{22}$$

Subsequently, $H^j(0,L)$, $j = 1,2$, denotes the usual Sobolev space of functions in $L^2(0,L)$ with derivatives up to order j also in $L^2(0,L)$. Let $L^\infty(0,+\infty; H^1(0,L))$ be the set of measurable functions $y: [0,+\infty] \to H^1(0,L)$ such that

$$|y(\cdot)|_{H^1(0,L)} \in L^\infty(0,+\infty).$$

Then $W^{1,\infty}(0,+\infty; H^1(0,L))$ denotes the space of functions $t \to y(t)$ in $L^\infty(0,+\infty; H^1(0,L))$ with first derivative with respect to t also in $L^\infty(0,+\infty; H^1(0,L))$.

Let H be the energy space associated to V given by (14)

$$H = \{(y,z,b,\eta) \in H^1(0,L) \times L^2(0,L) \times \mathbb{R} \times \mathbb{R};$$
$$b = y(L)\}. \tag{23}$$

It is easy to check that H is a Hilbert space for the scalar product

$$\langle X, \hat{X} \rangle = \int_0^L (z\hat{z} + ay_s\hat{y}_s)\,\mathrm{d}s + kb\hat{b} + \eta\hat{\eta}. \tag{24}$$

Then we define in H the domain of the unbounded linear operator A (see e.g. II.6. in Brezis (1983) for this notion)

$$\begin{aligned} Dom\,A = \{&(y,z,b,\eta) \in H^2(0,L) \times H^1(0,L) \times \mathbb{R} \times \mathbb{R}, \\ &y(L) = b,\ (ay_s)(0) = 0, \\ &z(L) + ((ay_s)(L) + kb)/K = \eta\}. \end{aligned} \tag{25}$$

We have the following result.

Lemma 2. *The operator A given by* (22)–(25) *is maximal monotone on H.*

Proof. From (18) and (19) we have

$$-\langle X, AX \rangle = \dot{V} \le 0, \quad \forall X \in Dom\ A. \tag{26}$$

A being linear, (26) is sufficient to prove that A is monotone. It all remains to check the range condition

$$R(I + A) = H. \tag{27}$$

The proof given in d'Andréa-Novel & Coron (1997) is quite similar to the one of Lemma 1 in d'Andréa-Novel et al. (1994). □

We can now state the following well-posedness result.

Theorem 1. (i) *For any initial data X_0 in Dom A, system* (20) *has a unique strong solution such that*

$$\begin{aligned} &X(t) = (y(t), y_t(t), b(t), \eta(t)) \in Dom\,A, \quad \forall t > 0 \\ &y \in W^{1,\infty}(0,+\infty; H^1(0,L)) \cap L^\infty(0,+\infty; H^2(0,L)). \end{aligned} \tag{28}$$

(ii) *For any initial data X_0 in H, system* (20) *has a unique weak solution X in H, given by $X(t) = S(t)X_0$, where $\{S(t)\}_{t \ge 0}$ is the semigroup of contractions on H generated by the operator A.*

(iii) *For any initial data X_0 in Dom A the functions*

$$\begin{aligned} &t \to \|X(t)\|_H, \\ &t \to \|(AX)(t)\|_H \end{aligned} \tag{29}$$

are nonincreasing.

Proof. A being a maximal monotone operator, Theorem 1 results for example from Theorem 3.1 in Brezis (1973). □

To be able to apply LaSalle's Invariance Principle, we have to check that the trajectories are precompact (see e.g. Dafermos & Slemrod, 1973; Slemrod, 1989). This precompactness is a corollary of the following lemma (its proof is quite similar to the proof of Lemma 2 in d'Andréa-Novel et al., 1994):

Lemma 3. *The canonical embedding from Dom A, equipped with the graph norm, into H is compact.*

3.3. Strong asymptotic stabilization

We prove the strong asymptotic stability of system (20) in H using LaSalle's invariance principle for the infinite dimension case (see Dafermos & Slemrod, 1973) and the Lyapunov function given by (14).

Theorem 2. *For any initial data X_0 in H, the solution $X(t) = S(t)X_0$ tends to zero in H:*

$$V(t) = \tfrac{1}{2}\|S(t)X_0\|_H^2 \to 0, \quad t \to +\infty. \tag{30}$$

Proof. We first consider the case

$$X_0 \in Dom\ A. \tag{31}$$

Using Theorem 1(iii), we have

$$\|X(t)\|_H + \|(AX)(t)\|_H \le \|X_0\|_H + \|AX_0\|_H. \tag{32}$$

From (32) and Lemma 3, the trajectory

$$\{X(t) = S(t)X_0,\ t \ge 0\} \tag{33}$$

is precompact in H, then the ω limit set $\omega(X_0) \subset Dom\, A$, is not empty and invariant w.r.t. to $S(t)$ (see Theorem 3.1 in Slemrod, 1989).

We now use LaSalle's invariance principle to show that $\omega(X_0) = \{0\}$.

Let $\bar{X}_0$ be an element of $\omega(X_0) \subset Dom\, A$, and let

$$\bar{X}(t) = (y, y_t, y(L), \eta)(t) = S(t)\bar{X}_0 \in \omega(X_0). \quad (34)$$

By applying LaSalle's invariance principle, let us consider the set

$$\frac{dV}{dt} = 0, \quad (35)$$

from (18) we deduce that

$$\begin{aligned} &\eta = y_t(L,t) + ((ay_s)(L,t) + ky(L,t))/K = 0,\\ &(ay_s)(L,t) + ky(L,t) = 0, \end{aligned} \quad (36)$$

which implies

$$\begin{aligned} &y_t(L,t) = 0,\\ &(ay_s)(L,t) + ky(L,t) = 0. \end{aligned} \quad (37)$$

Therefore, the function y is a solution of the following overdetermined system:

$$\begin{aligned} &y_{tt} - (ay_s)_s = 0,\\ &(ay_s)(0,t) = 0,\\ &(ay_s)(L,t) + ky(L,t) = 0,\\ &y_t(L,t) = 0. \end{aligned} \quad (38)$$

On the other hand, since $\bar{X}_0 \in Dom\, A$, from Theorem 1, y belongs to $W^{1,\infty}(0, +\infty; H^1(0,L)) \cap L^\infty(0, +\infty; H^2(0,L))$. We can then apply Lemma 3 in d'Andréa-Novel et al. (1994) to conclude that $y \equiv 0$, which implies $\bar{X}_0 = 0$ and then $\omega(X_0) = \{0\}$.

Using the density of $Dom\, A$ and point (ii) of Theorem 1, we end the proof by extending the result to any element X_0 in H. □

We now prove that the linear boundary feedback law given by (17), (16), (10) ensures the exponential decay of the energy V.

4. Exponential stabilization of the hybrid system

Let us introduce

$$Y = \begin{pmatrix} y \\ y_t \\ X_p \end{pmatrix}$$

and express the dynamics of subsystem (5) in the state-space form

$$\begin{cases} \dot{Y} = FY + Gv \text{ with} \\ FY = \begin{pmatrix} y_t \\ (ay_s)_s \\ 0 \end{pmatrix} \text{ and } G = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \end{cases} \quad (39)$$

We denote $\Phi(t, Y)$ the flow associated to subsystem (5) in closed-loop form with the stabilizing control $\bar{v}$ given by (10), and we introduce the following function:

$$Q(X) = W(Y) + \tfrac{1}{2}(\dot{X}_p - \bar{v})^2, \quad (40)$$

where

$$W(Y) = \int_0^{+\infty} E(\Phi(t, Y))\, dt. \quad (41)$$

It can be shown that W satisfies, $\forall X \in Dom\, A$:

$$\nabla W(Y)(FY + G\bar{v}) = -\|Y\|^2, \quad (42)$$

$\|\ \|$ being the norm associated to the energy E given by (6) (see e.g. Datko, 1970, for a proof of (42)).

Let us first establish the following lemma which links the time derivative of Q and the energy function V, when one uses the feedback law (17), (16), (10).

Lemma 4. *There exists $\nu > 0$ such that on Dom A*

$$\dot{Q} \leq -\nu V. \quad (43)$$

Proof. Computing the time derivative of Q along the trajectories of the hybrid system (1)–(2) in closed-loop form with (17), (16),(10) leads to

$$\dot{Q} = \nabla W(Y)(FY + G\bar{v}) + (\dot{X}_p - \bar{v})(U - \dot{\bar{v}} + \nabla W(Y)G).$$

Moreover, W being quadratic, there exists $\beta > 0$ such that

$$\|\nabla W(Y)G\| \leq \beta \|Y\|,$$

then from (16), (42) we can write

$$\dot{Q} \leq -\|Y\|^2 - \alpha(\dot{X}_p - \bar{v})^2 + \tfrac{1}{2}\|Y\|^2 + \frac{\beta^2}{2}(\dot{X}_p - \bar{v})^2$$

and then

$$\dot{Q} \leq -\tfrac{1}{2}\|Y\|^2 - \frac{2\alpha - \beta^2}{2}(\dot{X}_p - \bar{v})^2$$

and $\dot{Q} \leq 0$ for $\alpha > \beta^2/2$. Let us now define

$$\nu = \min(1; 2\alpha - \beta^2).$$

We have

$$\dot{Q} \leq -v\tfrac{1}{2}(\|Y\|^2 + (\dot{X}_p - \bar{v})^2),$$

which ends the proof of the lemma since

$$E = \tfrac{1}{2}\|Y\|^2 \quad \text{and} \quad V = \tfrac{1}{2}(\|Y\|^2 + (\dot{X}_p - \bar{v})^2). \qquad \square$$

We can now state our main exponential convergence result. This is the object of the following theorem.

Theorem 3. *If the gain α of the boundary feedback law U given by* (16) *satisfies* $\alpha > \max(K/2; \beta^2/2)$, *then there exist constants* $C_0 > 0$ *and* $\mu_0 > 0$, *independent of X solution of* (20), (21), *such that*

$$V(t) \leq C_0 V(0) e^{-\mu_0 t}, \quad \forall t \geq 0. \tag{44}$$

Proof. Lemma 4 allows us to show that the integral

$$\int_0^{+\infty} \|S(t)X_0\|_H^2 \, dt$$

converges for all X_0 in H, $S(t)$ being the contraction semi-group associated to the hybrid closed-loop system. More precisely, multiplying inequality (43) by -1, and integrating from 0 to $+\infty$, we obtain

$$\int_0^{\infty} \|X(t)\|_H^2 \, dt \leq Q(0)/v,$$

and this result is equivalent to the exponential convergence of the semi-group $S(t)$, as it has been shown by Datko (1970) in his Corollary. $\square$

Remark 4. In Lemma 6 of d'Andréa-Novel and Coron (1997), we had established a comparison result between Q and V which allowed us to conclude to the exponential convergence of V. But to establish this result, and more precisely the inequality $c_1 V \leq Q$ with $0 < c_1 \leq 1$, we had to use the following inequality (where E is the energy function associated to the *PDE*):

$$E(0) \leq \gamma \int_0^{+\infty} E(t) \, dt \quad \text{for a constant } \gamma > 0,$$

inequality proved in Lemma 1 of d'Andréa-Novel and Coron (1997), under the assumption that the gain $1/K$ of $\bar{v}$ given by (10) is sufficiently small.

Although this comparison lemma of d'Andréa-Novel and Coron (1997) is useless to prove exponential stabilization, it allows us to obtain more "quantitative" informations about the constant C_0 and the rate of convergence.

Furthermore, we did not succeed, for the moment, to prove the inequality $E(0) \leq \gamma \int_0^{+\infty} E(t) \, dt$ without the gain condition: $1/K$ sufficiently small.

5. Conclusion

In order to simulate system (1)–(2), we have used a modal decomposition and obtained a linear truncated finite

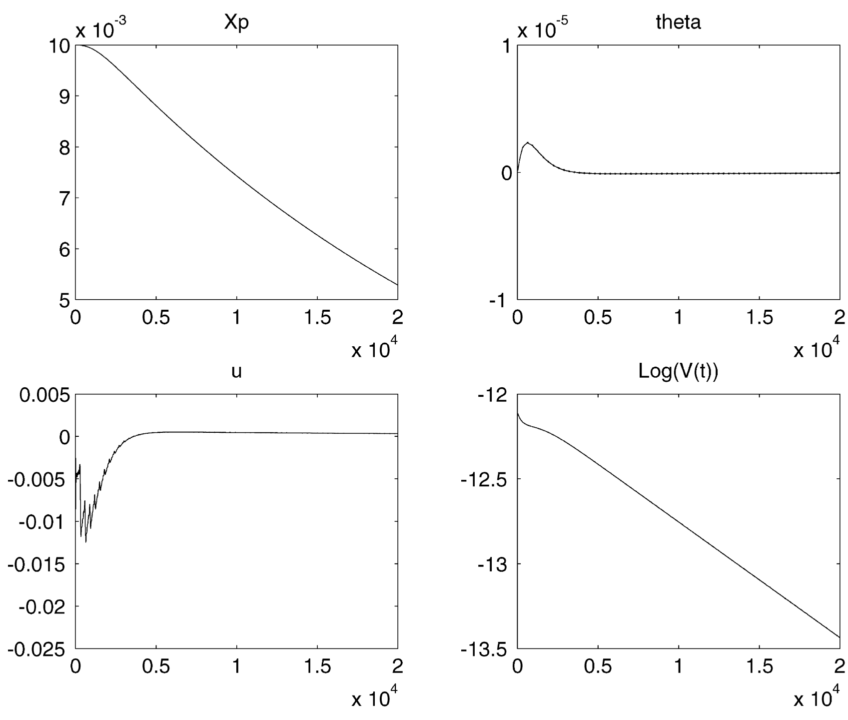

Fig. 2. X_p, θ, u and $\mathrm{Log}(V(t))$.

dimensional state formulation of the system as described e.g. in d'Andréa-Novel et al. (1994).

We have taken for u, the linear boundary feedback law given by (17). The linear truncated system in closed-loop form with this u has been integrated using a Matlab function. The results obtained with $K = 3$, $\alpha = 1.6$ and $k = 0.1$ are displayed in Fig. 2. The sampled period was equal to 1 ms and the simulation was performed during 20 s. At the top-left of Fig. 2 is given X_p and θ at the top-right. At the bottom-left the control u applied to the platform is plotted and Logarithm($V(t)$) at the bottom-right, to point out the exponential decay of the energy $V(t)$.

To conclude, we can see that the backstepping approach has been successfully applied to an hybrid PDE–ODE system, leading to an exponentially stabilizing boundary feedback controller. A work which is now under study, is the extension of our results in the case of a *variable length* flexible cable.

Acknowledgements

The authors wish to thank the anonymous reviewers for their valuable comments concerning the paper, and C.Z. Xu for useful discussions and for having mentioned to them reference Datko (1970).

References

d'Andréa-Novel, B., Boustany, F., Conrad, F., & Rao, B. P. (1994). Feedback Stabilization of a hybrid PDE–ODE System: Application to an overhead crane. *MCSS Journal*, *7*, 1–22.

d'Andréa-Novel, B., & Coron, J.-M. (1997). Exponential stabilization of an overhead crane with flexible cable via the cascade approach. *Proceedings of the IFAC SYROCO'97 Conference, Nantes*.

Byrnes, C. I., & Isidori, A. (1989). New results and counterexamples in nonlinear feedback stabilization. *Systems and Control Letters*, *12*, 437–442.

Brezis, H. (1983). *Analyse fonctionnelle*, *Théories et Applications*. Paris: Ed. Masson.

Brezis, H. (1973). *Opérateurs Maximaux Monotones et Semi-Groupes de Contraction dans les espaces de Hilbert*. Amsterdam: North-Holland.

Coron, J. -M., & d'Andréa-Novel, B. (1998). Stabilization of a rotating body-beam without damping. *IEEE Transactions on Automatic Control*, *43*(5), 608–618.

Coron, J. -M., & Praly, L. (1991). Adding an integrator for the stabilization problem. *Systems and Control Letters*, *17*, 89–104.

Dafermos, C. M., & Slemrod, M. (1973). Asymptotic behaviour of nonlinear contraction semi-groups. *Journal of Functional Analysis*, *13*, 97–106.

Datko, R. (1970). Extending a theorem of A.M. Liapunov to Hilbert space. *Journal of Mathematical Analysis and Applications*, *32*, 610–616.

Kanellakopoulos, I., Kokotovic, P. V., & Morse, A. S. (1992). A toolkit for nonlinear feedback design. *Systems and Control Letters*, *18*, 83–92.

Krstić, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Mifdal, A. (1997). Stabilisation uniforme d'un système hybride. *Comptes Rendus de Academie Sciences, Série I*, *324*, 37–42.

Morgül, O., Rao, B. P., & Conrad, F. (1994). On the stabilization of a cable with a tip mass. *IEEE Transactions on Automatic Control*, *39*(10), 440–454.

Morin, P., & Samson, C. (1997). Time-varying exponential stabilization of a rigid spacecraft with two control torques. *IEEE Transactions on Automatic Control*, *42*(4), 528–534.

Rao, B. P. (1993). Decay estimates of solutions for a hybrid system of flexible structures. *European Journal of Applied Mathematics*, *4*, 303–319.

Russell, D. L. (1975). Decay rates for weakly damped systems in Hilbert space obtained with control theoretic methods. *Journal of Differential Equations*, *19*, 344–370.

Slemrod, M. (1989). Feedback Stabilization of a Linear Control System in Hilbert Space with an a priori Bounded Control. *Mathamatics of Control*, *Signals and Systems, 2*, 265–285.

Tsinias, J. (1989). Sufficient Lyapunovlike conditions for stabilization. *Mathametics of Control, Signals and Systems*, *2*, 343–357.

Brigitte d'Andréa-Novel graduated from École Supérieure d'Informatique Électronique Automatique in 1984. She received the Doctorate degree from École Nationale Supérieure des Mines de Paris in 1987 and her Habilitation degree from Université Paris-Sud in 1995. She is currently Professor of Systems Control Theory and responsible for a research group on "Advanced Controlled Systems" at the Centre de Robotique-École Nationale Supérieure des Mines de Paris.

Her current research interests include nonlinear control theory and applications to underactuated mechanical systems, modeling and control of wheeled vehicles with application to automated highways and boundary control of flexible mechanical systems coupling ODE and PDE.

Jean-Michel Coron obtained the diploma of engineer from École Polytechnique in 1978 and from the "Corps des Mines" in 1981. He got his Thèse d'État in 1982. He was a successfull researcher at École Nationale Supérieure des Mines de Paris, Associate Professor at École Polytechnique, Director of research at CNRS and Director of the "Centre de Mathématiques et de Leurs Applications" (CNRS and ENS de Cachan). He is currently Professor at Université Paris-Sud.

His research interests include nonlinear partial differential equations, calculus of variations and nonlinear control theory.