“Sicut in caelo et in terra”

As in heaven and on earth

Data representations for seismic and gravitational waves

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Coalescence of two black holes  (credits: SXS)
DECOMPOSITION OF HARDY FUNCTIONS INTO SQUARE INTEGRLABLE WAVELETS OF CONSTANT SHAPE*

A. GROSSMANN‡ AND J. MORLET‡

Abstract. An arbitrary square integrable real-valued function (or, equivalently, the associated Hardy function) can be conveniently analyzed into a suitable family of square integrable wavelets of constant shape, (i.e. obtained by shifts and dilations from any one of them.) The resulting integral transform is isometric and self-reciprocal if the wavelets satisfy an “admissibility condition” given here. Explicit expressions are obtained in the case of a particular analyzing family that plays a role analogous to that of coherent states (Gabor wavelets) in the usual $L_2$-theory. They are written in terms of a modified $t$-function that is introduced and studied. From the point of view of group theory, this paper is concerned with square integrable coefficients of an irreducible representation of the nonunimodular $ax+b$-group.

1. Introduction.
1.1. It is well known that an arbitrary complex-valued square integrable function $\psi(t)$ admits a representation by Gaussians, shifted in direct and Fourier transformed

Collaboration with Jean Morlet with application to seismic data

I. Daubechies, Where do wavelets come from? A personal point of view
From earth- to spacetime- quakes

• **Seismic waves**
  
  density perturbations
  propagating as sound waves in
  the rock bed
  
  Produced e.g., in Earth-quakes or by
  man-made explosions for oil
  exploration

• **Gravitational waves**
  
  propagating space-time metric
  perturbations
  
  Consequence of General Relativity
  
  Produced by accelerated masses, e.g.,
  mergers of compact stars
“Cycle-octave” transform & oil exploration

Goupillaud, Grossman, Morlet, Cycle-Octave and related transforms in seismic signal analysis, Geoexploration, 1984

- Objective: higher resolution in the detection and localization of thin layers of oil reservoirs preserve *high frequencies* of the seismic echo
- Attenuation due to scattering by successive layers follows a constant-Q law large low freqs from small high freqs

- Standard data recording: low-pass filter for the “noise-dominated” high freqs
- Change to recording in **separate frequency bands** → 2D time-frequency representation
- Issues: size of data set/memory, signal recovery **discretization**, inversion
- Connected to coherent state rep of quantum mech.

"L-transform"

\[(L_g f)(x, y) = \frac{1}{\sqrt{c_g}} |y|^{-1/2} \int g \left( \frac{x - t}{y} \right) f(t) dt\]

Voice transf: **log scales**

\[y \rightarrow \pm 2^{-u}\]

Cycle-octave transf: **rescaled time**

\[x \rightarrow 2^u x\]

---

Fig. 4. Cycle-octave expansion: elementary wavelets.
Simulated signature of a sediment layer

Analysis

Reconstruction

Fig. 15. Voice representation of a synthetic seismic trace.

Fig. 16. Reconstruction (inverse transform) of Fig. 15.
From earth- to spacetime- quakes

- “Wavefront” reconstruction using a sensor array
- Spatial localization of the source of the wave
  - Timing measurement $\rightarrow$ importance of high frequencies
- Signal identification using time-frequency patterns

Similar signal analysis problems $\rightarrow$ similar solutions
A primer on gravitational waves
Les petits calculs d’un remaniement

Einstein avait raison

Rainer Weiss
LIGO/VIRGO Collaboration

Barry C. Barish
LIGO/VIRGO Collaboration

Kip S. Thorne
LIGO/VIRGO Collaboration

Alain Brillet

Thibault Damour
• Space-time is a **deformable and dynamical object**
• Gravity is a geometrical effect that emerges from space-time curvature

\[ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

« Spacetime tells matter how to move; matter tells spacetime how to curve 

JA Wheeler
Gravitational waves (1)

Einstein's equations $\rightarrow$ wave equation!
for small perturbations of flat space-time metric

There are waves of space and time

Those waves are transverse

Direction de propagation
General Relativity predicts **two polarizations**

Gravitational wave amplitude $h(t)$ has no dimension

$h \sim 0.5$
Black hole binary merger

inspiral  merger  ringdown

Chirp!

Characteristic waveform signature

\[ f_{GW} \approx 2f_{\text{orbital}} \]

\[ h \sim 10^{-21} \]
How do we detect gravitational waves?
Michelson interferometer
Michelson interferometer
\[ h = \frac{\delta \ell}{L} \sim 10^{-21} \]

\[ \delta \ell \sim 10^{-18} m \]

\[ L = O(1) \text{km} \]

Radius of atomic nuclei: \(10^{-15} m\) (x 1000)
LIGO Handford H1
LIGO Livingston L1
Virgo V1

~3000 km (10 light-ms)
~10000 km (30 light-ms)

Sensitivities during O2
H1: 60 Mpc
L1: 80 Mpc
V1: 25 Mpc

BNS range

Quasi-omnidirectional
(no pointing, see below)
2015
LIGO only

O1
4 months
~50 days
coincident

2016
LIGO only

2017
LIGO & Virgo

O2
Virgo
6 months
~100 days
coincident

2018
LIGO & Virgo

2019
LIGO & Virgo

O3
1 yr from Apr 2019

~5 TB of science data product
~15 millions segments
How do we search LIGO and Virgo data?
What do we search for?

Permanently:

- [Image]

Transiently:

- [Image]
What do we search for?

Permanent

Transient

With a model “parametric”

Matched filtering

Without a model “non-parametric”

Time-frequency Wavelets
Compact binary mergers

- We have accurate waveform models
  15 parameters: masses & spins + geometry
  To leading order, the phase is determined by
  \[ M = \frac{(m_1m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \]
  then by mass ratio, spins, ...
  - ... except in extreme region of parameter space,
    for instance: high-mass ratio, misaligned spins
    (→ precession), ...

Can be used as “template”

Correlate data with the templates (matched filtering)
Compact binary mergers
Matched filtering

Cover expected scenarios (parameter space coverage)
1 point = 1 waveform = 1 filter
400 000 filters!

Retain events that occur in coincidence on two or more detectors and come from the same filter

Main difficulties

Non-stationarity

Non-Gaussianity

“glitches” regularly appearing
Time-frequency searches

Transient signals = outliers in time-frequency representation
Time-frequency searches
Coherent WaveBurst (1)

Time-frequency map
- A collection of Wilson transforms at multiple scales
- Using the Meyer scaling function

Signal identification by pattern matching
- Clusters of “bright” time-frequency pixels
- General relativity is not required – Can detect unexpected sources

S. Klimenko et al., arXiv:1511.05999
Time-frequency searches
Coherent WaveBurst (2)

\[
\begin{bmatrix}
    x_H \\
    x_L \\
    x_V
\end{bmatrix}
= \begin{bmatrix}
    F_H^+ & F_H^x \\
    F_L^+ & F_L^x \\
    F_V^+ & F_V^x
\end{bmatrix}
\begin{bmatrix}
    h_+ \\
    h_x
\end{bmatrix}
+ \begin{bmatrix}
    n_H \\
    n_L \\
    n_V
\end{bmatrix}
\]

- **Network of detectors**
  - They all receive the same gravitational wave \( h_+ \) and \( h_x \)

- **Seek phase-coherent signals**
  - Likelihood → **Inverse problem**
    - Mixing: beampattern, propagation delays
  - Similar to beamforming (radar, radioastro)
    - Compensate time delays → Can be done efficiently in the time-frequency plane using Wilson transforms (orthonormal basis)
  - Source localization and waveform reconstruction using “bright” pixels
    - Small scales, high-frequency pixels are important!
Polarization reconstruction
- Bivariate signal $h_+$ and $h_x$

Polarized AM-FM signal
- Can be characterized by Stokes params (through quaternionic embedding)

See posters by F Feng & P Chainais
LIGO and Virgo observations so far...
GWTC#1

- 10 binary black hole mergers

- population hidden to conventional astronomy

Aug 17, 2017

Normalized amplitude

LIGO-Hanford

LIGO-Livingston

Virgo

Time (seconds)

Frequency (Hz)

Brightest
Lecture videos, Jupyter notebooks, challenge data set available online – Run on Google Colab cloud

Gravitational Wave Open Data Workshop #2

Tutorial 1.2: Introduction to GWpy

This tutorial will briefly describe GWpy, a python package for gravitational astrophysics, and walk-through how you can use this to speed up access to, and processing of, GWOSC data.

Click this link to view this tutorial in Google Collaboratory

This notebook was generated using python 3.7, but should work on python 2.7, 3.6, or 3.7.

Installation (execute the installation aire)

Note: we use pip, but it is recommended that you might look at a little different.

Gravitational wave
Open Data Workshop #2
Paris, April 8-10 2019

AstroParticule & Cosmologie
Paris Diderot University

Three-day workshop to learn how to access and analyze LIGO and Virgo data

http://www.gw-openscience.org

Science run O3

LIGO at ~120 Mpc (L1) and 100 Mpc (H1)
Virgo at ~50 Mpc (x ~2 wrt O2) ~90 % duty cycle
Very promising start! 14 alerts

https://gracedb.ligo.org/latest/
Science run O3 – current status

April 2019

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<th>90% loc.</th>
<th>Distance (Mpc)</th>
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May 2019

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BBH: binary black hole
BNS: binary neutron star
NSBH: neutron star black hole binary

Last BBH on Jun 02!
Prompt gravity signal from earthquakes

2011 Tohoku-Oki earthquake

Gravity signals could speedily warn of big quakes and save lives


[doi:10.1038/nature.2017.23045]