Hot plasma interactions
with radiofrequency waves
in the presence of fast particles

R. Dumont

CEA, IRFM, F-13108 Saint-Paul-lez-Durance, France.

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Electromagnetic waves in plasmas

Thomson scattering, Hard X-ray spectroscopy...
Interferometry, infrared

Electron cyclotron freq., reflectometry
Lower hybrid frequency

Ion cyclotron frequency

Turbulence

RF wave heating

Frequency [Hz]

$10^4$  $10^6$  $10^8$  $10^{10}$  $10^{12}$
Outline

▶ Radiofrequency (RF) waves in plasma
  ▶ Plasma heating by RF waves
  ▶ MHD (Alfvén) waves
  ▶ Fast particles in fusion plasmas

▶ Simulating low frequency waves
  ▶ Global simulation
  ▶ Variational approach to wave-field calculation
  ▶ The EVE code
  ▶ Benchmarks: MHD waves

▶ ICRF waves
  ▶ General features
  ▶ FLR expansion - Order reduction algorithm
  ▶ Simulation of ITER scenarios

▶ Quasilinear response

▶ Conclusions

▶ Prospects: beyond linear/quasilinear modeling
Advanced scenarios require electromagnetic waves excited by external antennas
- Ion and/or electron heating
- Non inductive current drive
- Rotation, flow drive (reduction/suppression of turbulence)
- Alpha-channelling

Three base ingredients of modelling
- Calculation of electromagnetic field: wave code
- Description of plasma response to the RF power: kinetic code
- Antenna / edge plasma interaction: antenna code

Room for improvement in each of these subtopics
Needs to work towards self-consistent loop
Electron Cyclotron Waves

- Frequency range:
  - $100\text{GHz} \lesssim f \lesssim 150\text{GHz}$

- Generators:
  - Gyrotrons

- General principle:
  - Cyclotron damping of ordinary/extraordinary wave by electrons, either thermal or superthermal.

- Main features:
  - Electron heating (localized)
  - Non-inductive current drive (localized)
  - (De-)stabilization of MHD modes (ST, NTM. . . )
Lower Hybrid Waves

- Frequency range:
  - $3\text{GHz} \lesssim f \lesssim 8\text{GHz}$

- Generators:
  - Klystrons, gyrotrons

- General principle:
  - Landau damping of toroidally asymmetric slow wave by superthermal electrons.

- Main features:
  - Non-inductive current drive (bulk)
  - Peripheral current drive (reactor)
  - Induced rotation
Ion cyclotron waves

- Frequency range:
  - $30\text{MHz} \lesssim f \lesssim 80\text{MHz}$

- Generators:
  - Tetrodes, diacodes

- General principle:
  - Cyclotron damping of fast wave by ions, either thermal or superthermal.
  - Landau damping of fast wave by electrons

- Main features:
  - Electron / ion heating
  - Non-inductive current-drive (central)
  - Induced rotation
ICRF heating of fusion plasmas

**ICRF**: Ion Cyclotron Range of Frequencies \( (f \sim 30 - 80\text{MHz}) \)

- **ICRF Power**
  - Fast Wave + Cycl. Res.
    - Fundamental absor.
    - Harmonic absorption
  - Fast Wave
    - ELD
    - TTMP
  - Ion Bernstein Wave
    - ELD

- Thermal ions
  - \( E < E_c \)
- Superthermal ions
  - \( E > E_c \)
- Thermal electrons

**Electron heating**

**Ion heating**
MHD waves

- Magnetically confined plasmas feature Alfvén waves (compressional / shear)
- Toroidal effects result in a coupling of these waves
- Within the frequency gaps lie global, regular, modes: **Alfvén Eigenmodes (AE)**
- These modes may be destabilized by fast ions (fusion born alphas / fast ICRF ions)

**Alfvén eigenmodes are crucial to ITER operation and performance**
Fast ions: \( v \gg v_{th} \equiv \sqrt{2k_b T_i/m_i} \)

Fast ions source in magnetic fusion plasmas:
- Neutral beam injection (NBI) ions
- ICRF accelerated ions
- Alpha particles (\( D + T \rightarrow \alpha + n \))

Low collisionality
- Relaxation time on electrons

\[
\tau_s \approx 4.20 \times 10^{14} \frac{A_i}{Z_i} \frac{T_{e,[eV]}^{3/2}}{n_e \ln(\Lambda)}
\]

- E.g. \( T_e = 10\text{keV}, \ n_e = 4 \times 10^{19} \text{m}^{-3} \rightarrow \tau_s \sim 1\text{s} \)
- Bounce time: \( \tau_b \sim 10^{-6} - 10^{-5} \text{s} \rightarrow \tau_b/\tau_s \ll 1 \)
1.9 MeV $\alpha$ particle in JET

Unperturbed orbit crucial for confinement
Banana orbits

Guiding-center trajectory of trapped ion

- Banana width:
  \[ \Delta_b \sim \frac{v_{\parallel 0}}{\omega_b} \]

- Confinement determined by
  \[ \Delta_b/a_0 \]

- Orbit effects are crucial when
  \[ \Delta_b \sim r \]

- E.g.: \( \alpha \) particle, \( E = 3.5\text{MeV} \)
  - JET: \( 2\Delta_b/a_0 \sim 0.8 \)
  - ITER: \( 2\Delta_b/a_0 \sim 0.2 \)
**Orbit effects**

**Toroidal number** \( N_\phi = +15 \)

Fast ion current inwards → **Co-current torque**

**Toroidal number** \( N_\phi = -15 \)

Fast ion current outwards → **Counter-current torque**

RF waves influence fast ions orbits and plasma toroidal rotation
Simulating low frequency waves
Full wave modeling

Wave equation

\[ \nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial \mathbf{j}_p}{\partial t} = \mu_0 \frac{\partial \mathbf{j}_{\text{ext}}}{\partial t} \]

Plasma current (space- and time-dispersive)

\[ \mathbf{j}_p(\mathbf{r}, t) = \int_{-\infty}^{t} dt' \int d^3\mathbf{r}' \bar{\sigma}(\mathbf{r}, \mathbf{r}', t, t') \cdot \mathbf{E}(\mathbf{r}', t') \]

Stationary and homogeneous plasmas:

\[ \bar{\sigma} = \bar{\sigma}(\mathbf{r} - \mathbf{r}', t - t') \]

\[ \mathbf{E}_{k,\omega} = \bar{\sigma}_{k,\omega} \cdot \mathbf{E}_{k,\omega} \]

Wave equation in Fourier space

\[ \mathbf{k} \times \mathbf{k} \times \mathbf{E}_{k,\omega} + \frac{\omega^2}{c^2} \left( 1 + \frac{i}{\omega \epsilon_0} \bar{\sigma}_{k,\omega} \right) \cdot \mathbf{E}_{k,\omega} = i\omega \mu_0 (\mathbf{j}_{\text{ext}})_{k,\omega}. \]
Full wave modeling (2)

- Stationary and homogeneous plasmas: plane waves

\[(E, B) = (E_0, B_0)e^{i(k \cdot r - \omega t)}\]

- Dielectric tensor, dispersion relation \(k = k(\omega)\), polarization...

- Stationary and weakly inhomogeneous plasmas: WKB formulation

\[(E, B) = (E_0(r, t), B_0(r, t))e^{i\psi},\]

- Slow time scale: \(E_0, B_0, \omega, k\)
- Fast time scale: \(\nabla \psi = k(r, t), \partial_t \psi = -\omega(r, t)\)
- Valid for plasmas varying over time-scale \(\Delta t\) and space-scale \(\Delta r\) if

\[\omega \gg 2\pi/\Delta t, \ k \gg 1/\Delta r\]

- Low frequency waves: \(\lambda \sim \Delta r\), cut-offs, resonances...

- WKB hypothesis does not hold
- Full wave solution required
Global modelling of LF waves

- **Wave excitation**
  - Current flowing in the antenna structure (frequency $\omega$)
  - Coupling is determined by the radiative resistance
    \[ P_{icrf} = R_{rad} I_{ant}^2 / 2 \]

- **Propagation**
  - Driven oscillation at frequency $\omega$
  - Space / time dispersion
  - Global electromagnetic field

- **Plasma response**
  - Modification of distribution functions
  - Collisional effects
Low frequency RF waves modelling: a summary

- Low frequency waves in fusion plasmas
  - Alvén eigenmodes
  - ICRF waves

- Three basic components of RF waves modelling

- Modelling of RF waves propagation and absorption in ongoing and future experiments should include
  - 2D / 3D effects
  - Non-thermal particle distributions
  - Finite orbit effects
Wave-field calculation: variational approach

- Electrical current / charge conservation

\[
\begin{cases}
    \mathbf{j}_{\text{ant}} + \mathbf{j}_{\text{part}} = \frac{1}{\mu_0} \nabla \times \nabla \times \mathbf{A} + \epsilon_0 \partial_t (\partial_t \mathbf{A} + \nabla \varphi) \equiv \mathbf{j}_{\text{maxw}}, \\
    \rho_{\text{ant}} + \rho_{\text{part}} = -\epsilon_0 \nabla \cdot (\partial_t \mathbf{A} + \nabla \varphi) \equiv \rho_{\text{maxw}}
\end{cases}
\]

- Three gauge-invariant functionals
  
  - Antenna functional
    \[
    \mathcal{L}_{\text{ant}} \equiv \mu_0 \int d^3 \mathbf{r} \left\{ \mathbf{j}_{\text{ant}} \cdot \mathbf{A}^* - \rho_{\text{ant}} \varphi^* \right\}
    \]
  
  - Maxwellian functional
    \[
    \mathcal{L}_{\text{maxw}} \equiv \mu_0 \int d^3 \mathbf{r} \left\{ \mathbf{j}_{\text{maxw}}(\mathbf{A}, \varphi) \cdot \mathbf{A}^* - \rho_{\text{maxw}}(\mathbf{A}, \varphi) \varphi^* \right\}
    \]
  
  - Plasma functional
    \[
    \mathcal{L}_{\text{part}} \equiv \mu_0 \int d^3 \mathbf{r} \left\{ \mathbf{j}_{\text{part}}(\mathbf{A}, \varphi) \cdot \mathbf{A}^* - \rho_{\text{part}}(\mathbf{A}, \varphi) \varphi^* \right\}
    \]
Wave-field calculation

- **Variational statement** \( \equiv \) extremalization of

\[
\mathcal{L}_{\text{part}}(\mathbf{A}, \varphi, \mathbf{A}^*, \varphi^*) + \mathcal{L}_{\text{maxw}}(\mathbf{A}, \varphi, \mathbf{A}^*, \varphi^*) + \mathcal{L}_{\text{ant}}(\mathbf{A}^*, \varphi^*)
\]

- **Decomposition of electromagnetic potential**: \((\mathbf{A}, \varphi) \equiv \sum_i \alpha_i (\mathbf{a}_i, \phi_i)\)

- **Decomposition of functionals**:

\[
\mathcal{L}(\mathbf{A}, \varphi, \mathbf{A}^*, \varphi^*) \equiv \mathcal{L}_{\text{part}} + \mathcal{L}_{\text{maxw}} = \sum_{ij} L_{ij} \alpha_i \alpha_j^*, \quad \mathcal{L}_{\text{ant}}(\mathbf{A}^*, \varphi^*) = \sum_j K_j \alpha_j^*,
\]

with

\[
L_{ij} \equiv \mathcal{L}(\mathbf{a}_i, \phi_i, \mathbf{a}_j^*, \phi_j^*), \quad K_j \equiv \mathcal{L}_{\text{ant}}(\mathbf{a}_j^*, \phi_j^*)
\]

- **Electromagnetic field calculation**

\[
\delta \left\{ \left[ (L_{ij,\text{part}} + L_{ij,\text{maxw}}) \alpha_i + K_j \right] \alpha_j^* \right\}_{\delta \alpha_j^*} = 0
\]

\(\rightarrow\) \((\mathbf{A}, \varphi)\) is obtained by solving \((L_{ij,\text{part}} + L_{ij,\text{maxw}}) \alpha_i = -K_j.\)
Energy balance and functionals

- Global energy balance
  - Power coupled by the antenna
    \[ \dot{W}_{\text{ant}} \equiv \left\langle \int d^3 r \, \mathbf{E} \cdot \mathbf{j}_{\text{ant}} \right\rangle = \frac{\omega}{2 \mu_0} \Im \left( \mathcal{L}_{\text{ant}} \right) \]
  - Power transferred to plasma species
    \[ \dot{W}_{\text{part}} \equiv \left\langle \int d^3 r \, \mathbf{E} \cdot \mathbf{j}_{\text{part}} \right\rangle = \frac{\omega}{2 \mu_0} \Im \left( \mathcal{L}_{\text{part}} \right) \]
  - Maxwellian functional
    \[ \mathcal{L}_{\text{Maxw}} = -2\mu_0 \int d^3 r \left( \frac{\varepsilon_0 |\mathbf{E}|^2}{2} - \frac{|\mathbf{B}|^2}{2\mu_0} \right) \text{ is a real quantity,} \]
    \[ \rightarrow \dot{W}_{\text{ant}} + \dot{W}_{\text{part}} = \frac{\omega}{2 \mu_0} \Im \left( \mathcal{L}_{\text{ant}} + \mathcal{L}_{\text{part}} \right) = 0 \]

- Local energy balance (Poynting theorem)
  \[ -i\omega \mathcal{W}_{\text{field}} + \mathcal{S}_{\text{Poynting}} - \dot{W}_{\text{ant}} - \dot{W}_{\text{part}} = \frac{i\omega}{2 \mu_0} \left\{ \mathcal{L}_{\text{Maxw}}(s) + \mathcal{L}_{\text{ant}}(s) + \mathcal{L}_{\text{part}}(s) \right\} \]
The \textbf{EVE} code

- **Physics features**
  - Wave equation formulated in terms of potentials
  - 2nd order Larmor radius code + Order Reduction Algorithm
  - Resolution of 2 ($E_\parallel = 0$) or 4 variables ($E_\parallel \neq 0$)
  - Uses quasi-local plasma functional

- **Numerical features**
  - 3D version functional (no coupling of toroidal modes)
  - Radial direction: finite elements (cubic + quadratic)
  - Fourier expansion in the toroidal and poloidal directions
  - Uses code generator for some parts
  - Core in Fortran 90 - Post-processing in Python

- **Objectives**
  - Main element of a wave + kinetic package
  - ICRF module for integrated modelling
  - Detailed physics studies of wave-particle interactions
Numerical implementation

- Spectral decomposition of state vector for periodic directions

\[ u_k(s, \theta, \phi) \equiv \sum_{mn} u_{kmn}(s) e^{i(m\theta + n\phi)} \]

- Finite elements in radial direction

\[ u_{kmn}(s) \equiv \sum_{jp} \alpha_{jmn}^{kp} h_p(s - s_j) \]

2\textsuperscript{nd} order FLR kinetic equations only involve \( \partial_s u \) and \( \partial_{ss}^2 u \)

\[ \rightarrow \text{ (} h_p \text{) are quadratic / cubic Hermite finite elements} \]
Numerical implementation (2)

- Variational principle yields directly Garlekin weak form
- Bilinear functionals

\[
\mathcal{L}_{\text{part}} + \mathcal{L}_{\text{Maxw}} = \mu_0 \int d^3r \left\{ \mathbf{j}(\mathbf{A}, \varphi) \cdot \mathbf{A}^* - \rho(\mathbf{A}, \varphi)\varphi^* \right\}
= \mu_0 \int ds d\theta d\phi J L_{eq}^{k\bar{k}} \times \mathbf{u}_k \mathbf{u}_k^*
= \mu_0 \delta_{n,\bar{n}} \int ds \left\{ J L_{eq}^{k\bar{k}} \right\} \bar{m} - m \mathbf{h}_{pj} \mathbf{h}_{\bar{p}j} \times \alpha_{jmn}^{kp} (\alpha_{jmn}^{k\bar{p}})^*
\]

- Provide stiffness matrix elements
- Toroidal modes (n) are decoupled (in axisymmetric devices)
- Poloidal modes (m) are coupled by equilibrium geometry and wave/particle interaction
Numerical implementation (3)

- **Antenna functional**

\[
\mathcal{L}_{ant} = \mu_0 \int d^3r \{ \mathbf{j}_{ant} \cdot \mathbf{A}^* - \rho_{ant} \varphi^* \}
\]

\[
= \mu_0 \int dsd\theta d\phi JL_{ant}^{k\bar{k}} \times u_{k}^*
\]

\[
= \mu_0 \int ds \{ JL_{ant}^{k\bar{k}} \}_{\bar{m},\bar{n}} h_{\bar{p}j} \times (\alpha_{j\bar{m}\bar{n}}^{k\bar{p}})^*
\]

- Provides source vector (RHS)
- Antenna current is decomposed in toroidal harmonics:

\[
\mathbf{j}_{ant} = \sum_{n} j_{ant,n} e^{in\phi}
\]

- Separate computation for each \( n \)
- 3D solution is constructed afterwards
Numerical implementation (4)

- Electromagnetic field calculation
  - Linear system solution (block matrix)
    - Matrix elements calculation:
      - Fast Fourier Transforms (FFTW)
      - Radial integrals (Gauss quadrature)
    - Code is partially generated by symbolic manipulation software
    - Boundary conditions and unicity directly applied to stiffness matrix
    - Parallel matrix inversion (ScaLAPACK)
K. Appert and J. Vaclavick


- Homogeneous plasma column
  - Hydrogen plasma
  - $n_e = 0.52 \times 10^{19} \text{m}^{-3}$
  - $B_0 = 1 \text{T}$
  - No eq. plasma current

- Linearized MHD model

\[
\begin{aligned}
\rho \partial_t \delta \mathbf{v} &= \delta \mathbf{j} \times \mathbf{B}_0 \\
\delta \mathbf{E} + \delta \mathbf{v} \times \mathbf{B}_0 &= m_i (\delta \mathbf{j} \times \mathbf{B}_0)/(e \rho) \\
\nabla \times \delta \mathbf{B} &= \mu_0 \delta \mathbf{j} \\
\nabla \times \delta \mathbf{E} &= -\partial_t \delta \mathbf{B}
\end{aligned}
\]

- Perturbed quantities $\propto \exp(i(k_z z + m \theta - \omega t))$
  - Dispersion relation $D_{m,a_p,a_v}(\omega/\omega_{ci}, k_z) = 0$
- **Alfvén frequencies:** $\omega < \omega_{cH}$
  - Divergence free helix antenna
    - Single poloidal mode ($m = -1$)
    - $k_\parallel = 15m^{-1}$

Peaks in the antenna loading are obtained at the Alfvén wave eigenfrequencies
EVE benchmarks: MHD waves

- Fast wave frequencies: $\omega > \omega_{cH}$
  - Divergence free helix antenna
    - Single poloidal mode ($m = -1$)
    - $k_\parallel = 15 \text{m}^{-1}$

![Dispersion relation](image)

![Frequency scan (EVE)](image)

Peaks in the antenna loading are obtained at the fast magnetosonic wave eigenfrequencies
ICRF waves
Plasma heating by ICRF waves

- **ICRH** (= Ion Cyclotron Resonance Heating)
- **General principle:**
  - A (fast magnetosonic) wave is excited on the Low Field Side of the tokamak
  - Its frequency is in the range $\omega \sim \Omega_{ci}$ ($f \approx 50\text{MHz}$)
  - Resonance relation: $\omega = p\Omega_{ci} + k_\parallel v_\parallel$, $p$ integer
  - In most scenarios, it is used for ion heating
  - Creation of superthermal populations (fast ions)
D(H) minority heating scenario

- \( R_0 = 3 \text{m}, \ a_0 = 0.8 \text{m}, \ B_0 = 3.1 \text{T} \)
- \( n_{e0} = 5 \times 10^{19} \text{m}^{-3} \)
- \( n_{H,b}/n_e = 4.75\% \)
- \( n_{H,f}/n_e = 0.25\% \)
- \( T_{e0} = T_{D0} = T_{H,b0} = 5 \text{keV}, \ T_{H,f0} = 10 \text{keV} \)

Strap antenna

- \( f_{FW} = 46 \text{MHz} \)
- \( N_{\text{tor}} = 15, \ k_{||,\text{ant}} \approx 4 \text{m}^{-1} \)
- Current perp. to \( B_0 \)

Computation grid

- \( N_s = 250 + 50, \ N_\theta = 128 \)
- 49 poloidal modes
- 1 toroidal mode
**Ion Bernstein wave:**
- excited by mode conversion
- small wavelength
- damps on thermal electrons
- backward wave

**Fast Magnetosonic wave:**
- direct excitation by the antenna
- large wavelength
- damps on ions / electrons
- forward wave

**Ion cyclotron layer**
(Fundamental Hydrogen)

**Mode conversion layer**
(from cold plasma theory)
Wave equation

- System driven at frequency $\omega$:

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2} \left( \mathbf{E} + \frac{i}{\omega \epsilon_0} \mathbf{j} \right) = i\omega \mu_0 \mathbf{j}_{\text{ant}}$$

Non-local response

$$\mathbf{j}(\mathbf{r}) = \sum_s \int d^3 \mathbf{r}' \quad \underline{\sigma}_s(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')$$

Conductivity kernel

Finite Larmor radius expansion

$$\mathbf{j}(\mathbf{r}) = \sigma_s^{(0)} \cdot \mathbf{E}(\mathbf{r}) + \sigma_s^{(1)} \cdot (\mathbf{r}_c \cdot \nabla) \mathbf{E}(\mathbf{r}) +$$

$$\frac{1}{2} \sigma_s^{(2)} \cdot (\mathbf{r}_c \cdot \nabla)^2 \mathbf{E}(\mathbf{r}) + \frac{1}{6} \sigma_s^{(3)} \cdot (\mathbf{r}_c \cdot \nabla)^3 \mathbf{E}(\mathbf{r}) \ldots$$

Expansion parameter:

$$|\mathbf{r}_c \cdot \nabla| \sim k_\perp \rho_i$$
FLR vs all-orders codes

► All-order codes (METS, AORSA, ...): $k \perp \rho_i$ arbitrary
  ▶ Spectral treatment: $E \propto \exp(i k \perp \cdot r)$
  ▶ Non-local current:

$$j(r) = j(r_{gc}) \sum_{p=-\infty}^{\infty} J_p(k \perp \rho_i) e^{ip\phi_c}$$

  ▶ Include all cyclotron harmonics
  ▶ Dense matrices $\rightarrow$ huge codes

► FLR codes (EVE, TORIC, ...): $k \perp \rho_i \ll 1$
  ▶ Retain contributions up to $\rho_c^2$

$$j(r) = \sigma_s^{(0)} \cdot E(r) + \sigma_s^{(1)} \cdot (r_c \cdot \nabla) E(r) + \frac{1}{2} \sigma_s^{(2)} \cdot (r_c \cdot \nabla)^2 E(r)$$

  ▶ Limited to cyclotron harmonics $p = -2 \ldots 2$
  ▶ Second order wave equation $\rightarrow$ sparse matrix problem
Order Reduction Algorithm

- The field is treated spectrally in the perp. direction
  \[ \mathbf{E} \propto \exp(i\mathbf{k}_\perp \cdot \mathbf{r}) \]

- \( k_\perp \) is obtained from the local fast wave dispersion relation

- Allows to retain all IC cyclotron harmonics
- No mode conversion
Parasitic absorption at third harmonic by D ions ($\omega \sim 3\Omega_{cD}$)

EVE predicts correct 3D damping with Order Reduction Algorithm
Artificial IBW damping

Electric field (equatorial plane)

- With add. damping
- No add. damping

Artificial damping of the IBW
- Eases the convergence
- Does not affect the power split (for these parameters)
**3D effects: antenna**

- Oscillating current flowing in straps with relative phase shifts
  - \( I_0 \propto \exp(i(kz_0 - \omega t)) \)
  - \( I_1 \propto \exp(i(k(z_0 + \Delta z) - \omega t + \varphi_1)) \)

- Antenna current is decomposed in toroidal harmonics
  \[
  j_{ant}(\phi) = \sum_n j_{ant,n} e^{in\phi}
  \]

- 3-D field and composite power deposition are reconstructed afterwards
  \[
  E[V/m] = w_p \sum_n \sigma_n \bar{E}_n e^{in\phi},
  \]
  \[
  \rho_{abs}[W/m^3] = w_p^2 \sum_n |\sigma_n|^2 \bar{p}_{abs,n}
  \]
Adjustable phase shifts yield flexible antenna phasing.
Prescribed profiles by ITPA benchmarking activity

- Adding a small fraction of $^3$He ions in ITER plasmas
  - improves the wave damping
  - increases the fusion reaction rate
Prescribed profiles by ITPA group

Flexible antenna phasing allows to drive central non-inductive current with weak impact on the power split between plasma species.
Modeling the quasilinear plasma response
Example: FWEH in JET

- Fast Wave: general features
  - Cold propagation
  - Belongs to the same branch as the compressionnal Alfvén wave

- Fast Wave Electron Heating
  - Electron damping by ELD + TTMP
  - Absorption is very sensitive to $\beta_e$

- A FWEH scenario in JET
  - 50% Hydrogen - 50% Deuterium
  - $B_0 = 1.34T$, $n_{e0} = 2.5 \times 10^{19} m^{-3}$
  - Wave frequency: $f_{FW} = 48$MHz
  - Parallel wavenumber: $k_{||,ant} \approx 8m^{-1}$
  - H-2 cyclotron layer on the HFS
FWEH in JET: Parasitic ion absorption

Power deposition profiles

- Electrons
  - $T_H(0) = 4$ keV
  - $T_H(0) = 30$ keV

- Hydrogen

Power split between species

- Hydrogen
- Electrons

- Competing electron / ion damping
  - Ion damping increases with $T_H$
  - Bootstrapping process
  - Self-consistent wave + kinetic simulation required
Kinetic equation for $f_s$:

$$\frac{∂f_s}{∂t} + \mathbf{v} \cdot \frac{∂f_s}{∂\mathbf{r}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{∂f_s}{∂\mathbf{v}} = \left( \frac{∂f_s}{∂t} \right)_{\text{coll}}$$

Linearization: $f_s(\mathbf{r}, \mathbf{v}, t) \equiv f_{s,0}(\mathbf{r}, \mathbf{v}) + \delta f_s(\mathbf{r}, \mathbf{v}, t)$

**Linear treatment** (wave-field calculation): $(Δt)_w \sim 2\pi/ω$

- $f_{s,0}$ stationary
- Collisions are neglected

**Quasilinear treatment** (response calculation): $(Δt)_q \ll 2\pi/ω$

- $f_{s,0}$ varies slowly compared to $2\pi/ω$ (secular effects)

$$\langle δf_s(t) \rangle = 0 \quad \rightarrow \quad \langle f_s(t) \rangle = f_{s,0}(t)$$

where $\langle \cdot \rangle$ means averaging over fast time scale

- Collisions need to be included
Mixed linear / quasilinear simulation

- Full-wave calculation
  - Prescribed Maxwellian distribution function
  - Yields: electromagnetic field
  - Yields: collisionless power absorption

- Kinetic calculation
  - Prescribed electromagnetic field
  - Yields: modified distribution functions
  - Yields: collisional redistribution of power

Wave-field calculation

Quasilinear calculation
Integration with kinetic calculation

- Wave calculation
  - Variational approach
    \[ \sum_s \mathcal{L}_{\text{part},s} + \mathcal{L}_{\text{Maxw}} + \mathcal{L}_{\text{ant}} = 0, \]
  - Resonant plasma functional
    \[ \mathcal{L}^{(\text{res})}_{\text{part},s} = \mu_0 \sum_{N_1=p, N_2, N_3=N} (2\pi)^3 \int d^3 \mathbf{J} \frac{\omega}{\omega - N_i \Omega_i} \frac{\partial f_{s,0}}{\partial J_i} |\delta h_{p, N_2, N}|^2 \]

- Kinetic response calculation
  - Fokker-Planck equation
    \[ \frac{\partial f_{s,0}}{\partial t} = \hat{C} f_{s,0} + \frac{\partial}{\partial J_i} D_{ij}^{(QL)} \frac{\partial f_{s,0}}{\partial J_j} \]
  - Wave quasilinear diffusion coefficient
    \[ D_{ij}^{(QL)} = 2\pi \sum_{N_1=p, N_2, N_3=N} N_i N_j |\delta h_{p, N_2, N}|^2 \delta(\omega - N_i \Omega_i) \]
Quasilinear response: a minimal model

- Fokker-Planck equation (orbit averaged)

\[
\frac{\partial f_{s,0}}{\partial t} = \langle C(f_{s,0}) \rangle + \langle D_w(f_{s,0}) \rangle.
\]

- **EVE** provides the **orbit averaged** wave QL diffusion coefficient

  \[ \rightarrow \text{Self-consistent framework [R. Dumont, Nucl. Fusion 2009]} \]

- **Quasi-local, uniform** approximation [Stix, 1992]:

\[
D_{V\perp V\perp}^{(QL)}(v, \psi) = D_0 \sum_{\theta_{\text{res}}} |\delta A_{+} J_{p-1}(k_{\perp} v_{\perp} / \Omega_{cs}) + \delta A_{-} J_{p+1}(k_{\perp} v_{\perp} / \Omega_{cs})|^2.
\]
Iterative evaluation of QL diffusion coefficient

- Quasilinear deformation for the heated species (i)

\[
\frac{\partial f_i}{\partial t} \approx \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp D_{v_\perp v_\perp}^{(QL)} \frac{\partial f_i}{\partial v_\perp} + \left( \frac{\partial f_i}{\partial t} \right)_c
\]

- First guess for the value of \( D_0 \):

\[
p_{abs,lin} = \int d^3v \frac{m_i v^2}{2} \left\{ \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp D_{v_\perp v_\perp}^{(QL)} \frac{\partial f_{\text{max}}}{\partial v_\perp} \right\}
\]

provided by EVE

- **Numerical solution** of the FP equation \( \rightarrow f_i \)

- Quasilinear power dissipation:

\[
p_{abs,qlin} = \int d^3v \frac{m_i v^2}{2} \left\{ \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp D_{v_\perp v_\perp}^{(QL)} \frac{\partial f_i}{\partial v_\perp} \right\}
\]

- \( D_0 \) is rescaled until \( p_{abs,lin} = p_{abs,qlin} \).

- Iterative process
Local collision operator

- Relaxation on **Maxwellian background** \((\mu \equiv v_\parallel / v)\)

\[
\left( \frac{\partial f_i}{\partial t} \right)_c = \sum_{s \neq i} \left\{ \frac{1}{v^2} \frac{\partial}{\partial v} v^2 \left[ \left( D^{i/s}_{vv} \frac{\partial f_s}{\partial v} \right) - F^{i/s}_v f_s \right] + \frac{1}{v^2} \left[ \frac{\partial}{\partial \mu} D^{i/s}_{\mu \mu} \frac{\partial f_s}{\partial \mu} \right] \right\}
\]

Friction + diffusion

Pitch-angle scattering

- Power **transferred to background species** by collisions:

\[
p_{\text{coll}} = \sum_s p^{i/s}_{\text{coll}} = \int d^3v \frac{m_i v^2}{2} \left( \frac{\partial f_i}{\partial t} \right)_c
\]

- Fokker-Planck coefficients (**Energy**):

\[
D^{i/s}_{vv} \equiv \Gamma^{i/s}_{v} \frac{\Psi(v/v_{th,s})}{2v}, \quad F^{i/s}_v \equiv -\frac{\Gamma^{i/s}_v}{v^2} \frac{m_i}{m_s} \Psi(v/v_{th,s})
\]
For fast ions, the main collisional processes are:

- ![Pitch-angle scattering (by bulk ions)](image)
- Collisional friction (on electrons)
- Energy diffusion (on electrons)
Plasma parameters
- $B_0(0) = 3.6\,\text{T}$,
- Deuterium + 6.0% Hydrogen,
- $n_{e0} = 6 \times 10^{19}\,\text{m}^{-3}$
  ($n_l = 4.2 \times 10^{19}\,\text{m}^{-3}$),
- $T_{e0} = 3.2\,\text{keV}$, $T_{i0} = 2.8\,\text{keV}$

Wave parameters
- $f = 57\,\text{MHz}$, $P_{RF} = 3.5\,\text{MW}$
- Dipole phasing

Calculation parameters
- 100 radial points
- 33 poloidal modes
- 10 toroidal modes
Fast population build-up

**Convergence**

- Final state is usually attained after 5-10 iterations
Collisional power redistribution

Relaxation $H \rightarrow$ background ions

- Thermal power sources 
  ($\eta_H$ : power fraction to H ions)
  - $p_e = p_{e,eve} + \eta_{H,eve} \times p_{H/e}$
  - $p_i = p_{D,eve} + \eta_{H,eve} \times p_{H/d}$
  - $|p_{H/H}| \ll |p_{H/D}|, |p_{H/e}|$

Composite power deposition profiles

- Electrons: 44.5%
- Ions: 52.4%
From ion to electron heating

- Electron heating usually dominant in D(H) minority regime.

![Graph showing power fraction to electrons and ions vs. RF power]

- $T_f = 75\text{keV}$, $T_f = 101\text{keV}$, $T_f = 164\text{keV}$, $T_f = 206\text{keV}$, $T_f = 243\text{keV}$

- $W_{\text{fast}} / W_{\text{dia}}$ [%] vs. $P_{\text{RF}}$ [MW]

- $W$ [MJ] vs. $P_{\text{RF}}$ [MW]
State-of-the-art RF modelling

- A physics well understood
  - Wave-field modelling
    - Full wave codes (MHD, ICRH, LHCD)
    - Ray-tracing codes (LHCD, ECRH)
  - Quasilinear response calculation
    - Simplified bounce-averaged models (AQL, CQL3D...)
    - Orbit following or Orbit averaged Monte Carlo solvers (FIDO, ITM...)
  - Sill room for improvement
    - Improving integration of wave / quasilinear calculation
    - Non-Maxwellian distributions $f_{s,0}(E, \Lambda, P_\phi)$ as inputs
    - Order reduction algorithm for $k_\perp \rho \gtrsim 1$, higher harmonics...
    - 3D equilibria (stellarators)

- Beyond mixed linear / quasilinear models
  - Limitations
    - Driven problem at given frequency $\omega$
    - No non-linear wave-coupling
    - No self-consistent treatment of turbulence
  - Towards inclusion in non-linear codes
Beyond linear modelling
MHD + Kinetic simulation of LF waves

- **Kinetic (Vlasov) equation**
  - **Linearization**
    - Dielectric tensor / Particle Lagrangian
      - Full-Wave codes
        - LEMAN, EVE
  - **Fluid moments**
    - Fluid (MHD) equations (Linear / Non linear)
      - Extension to hybrid kinetic-MHD models
        - XTOR-K, HMGC
  - **Simplified Lagrangian**
    - Gyrokinetic theory (Linear / Non linear)
      - Gyrokinetic MHD
        - LIGKA
Fast particles in gyrokinetics: EGAMs

- GAMs (Geodesic Acoustic Modes)
  - naturally present in toroidal geometry
  - induce flows which can reduce turbulence
  - damped by Landau effect $\gamma_L \sim -\exp(-q^2)$
  - can be excited by fast ions ($\rightarrow$ EGAMs)

- Numerical study of EGAMs
  - Excitation condition: $\partial_E F_{eq} |_{v_{\parallel, \text{res}}} > 0$
    $\rightarrow$ non linear kinetic mode
    $+ \text{ self-consistent description of turbulence}$
    $\rightarrow \text{ heat source added in GYSELA}$
  - GK equation + quasineutrality [Brizard, 2007]

$$B_{\parallel}^* \partial_t F + \nabla \cdot (B_{\parallel}^* \dot{x}_G F) + \partial_{v_{G,\parallel}} (B_{\parallel}^* \dot{v}_{G,\parallel} F) = C(F) + S_{\text{bulk}} + S_{\text{fp}}$$

- Adiabatic electrons
- $C(F)$ collisions operator [Dif-Pradalier, 2011]
- $S_{\text{bulk}}$ bulk heating (flux-driven simulations) [Sarazin, 2011]
- $S_{\text{fp}}$ fast particles energy source [Zarzoso, 2011]


**Fast particles source implementation**

- **EGAM excitation** \( \leftrightarrow \partial E F_{eq} | v_\parallel = v_{res} > 0 \leftrightarrow \text{Resonance in } v_\parallel. \)

- **Bulk energy source**
  \[
  \partial E F_{eq} | v_\parallel = v_{res} < 0
  \]

- **Fast particles source**
  \[
  v_0 \neq 0 \rightarrow \partial E F_{eq} | v_\parallel = v_{res} > 0
  \]

- \( S_{bulk} \) and \( S_{fp} \) do not inject particles.
Comparing simulations with/without EGAMs

▶ Two flux driven simulations $S = S_{\text{bulk}} + S_{fp}$. Only difference: $S_{fp}$ with $v_0 = 0$ and $v_0 = 2$. Same heating power.

▶ Total heating such that $\nabla T \equiv \int d^3 v (E S(v))/\chi_{\text{neo}} < \nabla T_{\text{crit}}$ → No expected ITG turbulence.

▶ Parameters

- $\rho^* = 1/64$ ($\rho^*_{\text{ITER}} = 2 \cdot 10^{-3}$), $\nu^* = 0.1$ (banana regime)
- $N_r = 128$, $N_{\theta} = 128$, $N_{\varphi} = 64$, $N_{v_\parallel} = 128$, $N_\mu = 16 \Rightarrow N_{\text{proc}} = 512$
New source successfully excites EGAMs

- When $v_0 = 0$: $\partial E F_{eq} < 0 \rightarrow$ Landau damped GAMs.
- When $v_0 = 2$: $\partial E F_{eq} > 0 \rightarrow$ EGAMs excited at $\omega_{EGAM} \approx \omega_{GAM}/2$.

Work in progress. Prospects:
- Study turbulent transport of fast ions
- Detailed study of mechanisms responsible for GAM saturation
- Development of a source directly based on $D_{QL}(v)$ from wave code