PDE/ODE models of collective swimming: viscosity, interactions and collisions.

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Abstract

We present a review of our work on PDE models of swimming bacteria.

- *Analytical study of dilute suspensions.* We introduced a stochastic PDE model for a dilute suspension of self-propelled prolate spheroids with tumbling and obtained an explicit asymptotic formula for the effective viscosity (E.V.) that explains the mechanisms of the drastic reduction of E.V.

- *Analytical and numerical study of dilute and semi-dilute bacterial suspensions.* We introduced a semi-dilute model for swimming bacteria that includes pairwise interactions and obtained an explicit asymptotic formula for the E.V. We also conducted numerical modeling of a large number of interacting bacteria with excluded volume constraints. Comparison with the dilute case leads to a phenomenon of stochasticity arising from a deterministic system.

- *Analytical and numerical studies based on kinetic models: beyond Mean-Field.* We seek to capture a phase transition in the bacterial suspension – an appearance of correlations and large scale structures with an increase of concentration. Collisions of the bacteria, ignored in most of the previous works, play an important role in this study, which is based on the kinetic theory approach. We introduced an analytical model of two swimmers, and show that collisions must be modeled by the Navier boundary conditions (with slip) rather than the commonly used no-slip.

Collaborators: PSU students S. Ryan and B. Haines, and V. Gyrya, PSU postdoc M. Potomkin, P-E. Jabin (Maryland), and DOE scientists I. Aronson and D. Karpeev (both Argonne Nat. Lab).