PhD thesis proposal in applied mathematics


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Doctorale School: MIIS - Mathématiques, Information, Ingénierie des Systèmes (Normandie)

Fellowship duration: 36 months

1. Context and motivation

Optimal control concerns the determination of control strategies for complex systems, with the aim of optimising their performances and making them evolve according to well-defined objectives (reaching a target, avoiding obstacles or observers, etc). This field of research was born in the 60s, motivated by the “space race” and the need to develop a new theory and new computational methods for the determination of flight paths in space exploration. The field now has a much broader scope than what early applications to aerospace engineering would suggest, and now encompasses applications where the state system describe challenging scientific and technological phenomenon with social, economic and environmental impacts of great importance (in neuroscience, climate modelling, geophysics, chemistry, financial mathematics, etc.).

The foundations of the field of optimal control theory are now well established and have benefited from several important contributions developed in recent decades using different mathematical tools (geometric control, optimisation theory, variational analysis, partial differential equations, numerical analysis, etc.). However, the new applications coming from more complex technologies require additional developments, and calling on modern tools, in order to be able to consider concrete problems involving nonlinear complex systems submitted to uncertainties of the model or the environment.

Stochastic partial differential equations (SPDEs) are the most adequate modern mathematical tool for modelling many biological, physical and economic systems subjected to the influence of noise, whether intrinsic (modelling uncertainties, random initial conditions...) or extrinsic (environmental influences, random forcing, …). In many cases, the presence of noise leads to new physical behaviours and new mathematical properties. SPDEs have become an important field in mathematics, at the intersection of probability theory and analysis of partial differential equations. Many significant advances have been achieved in recent years leading to the development of a rigorous general theory that provides a precise meaning to the notion of solutions and also an adequate framework for analysing numerical algorithms devoted to the approximation and computation of these solutions.
2. Objectives

The subject of this thesis concerns the development of new theoretical and numerical approaches for some control problems governed by partial differential equations.

Control theory of SPDEs have already been studied in many contributions. Several results have been established regarding the exact, null or approximate controllability properties for various stochastic equations (of transport, parabolic or hyperbolic type). An extensive literature have been also devoted to the stability analysis of SPDEs providing valuable insights into the regularity and global behaviour of the solutions of SPDEs.

Besides, new mathematical tools have been introduced to extend the theory of optimal control problems governed by SPDEs. Necessary or sufficient optimality conditions have been derived for a class of control problems, providing important information on the nature of optimal controls, in particular, for situations in which optimal controls can be realised by a feedback control law.

Many practical optimal control problems involve constraints on state variables, to avoid some regions of the state space where perhaps operation is unsafe or the dynamic models considered are no longer valid. The current contributions on control of SPDEs do not take account of such constraints. The understanding of state-constrained optimal controls is in many important respects incomplete, and important questions concerning the structure of optimal state constrained trajectories, higher order conditions, conditions for local optimality and efficient computational methods are still open.

The aim of this project is to further develop the fundamental theory of optimal control problems of SPDEs under end-point or/and distributed state constraints, and also to provide computational schemes for their solutions.

Some references


3. **Background / Required selection criteria:**

The application requirement is a Master degree with strong background in applied mathematics, in particular in probability theory and/or in analysis of PDEs.

Some knowledge in control theory and/or Optimization and/or numerical analysis will be appreciated.

Some knowledge of programming skills (C++, matlab, python etc) is required.

Good written and oral English language skills are required.

**Personal characteristics:**
- Motivation for conducting research at an advanced level
- Good communication skills
- Ability to work independently and in a team

The application should be accompanied by a detailed CV, a motivation letter, and the name of two referees (to whom a recommendation letter will be requested, if the candidate is shortlisted).

Applications will be reviewed as they are received until June 1st, or until a suitable candidate is found.

4. **Funding/Context:**

The PhD candidate will be part of the group “Optimization, Control” in the Applied Mathematics Laboratory - LMI - at INSA Rouen Normandie.

The laboratory provides a rich and stimulating environment for demanding and high-level research in applied mathematics combining theoretical subjects and challenging applications. In particular, the PhD thesis is part of a research project “Chaire d’Excellence - COPTI”, lead by Prof. Hasnaa Zidani, and funded by Région Normandie.

The gross salary is about (~1800 euros per month), the PhD student have also the possibility to apply for a teaching position (raising the salary).