<u>**Summary of PhD proposal**</u>

Output feedback and observer design in PDE setting over sampled-in-time/space measurements – Experimentations and validation for thermal processes

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Summary: *Being among modern control trends (see [1] for recent investigations), output feedback design of diffusion systems with distributed-in-space parameters is further developed based on available measurements sampled in both time and spatial variables. The approach to be developed relies on the semigroup representation of the solutions of parabolic (diffusion) PDEs and convert the observer design problem over sampled measurements to the standard one over continuous-time measurements. Robustness issues of the proposed observer-based feedback design are additionally to be investigated in the presence of external disturbances, affecting the plant dynamics, and measurement noise. Capabilities of the resulting synthesis are expected to be supported not only in numerical but also experimental study.*

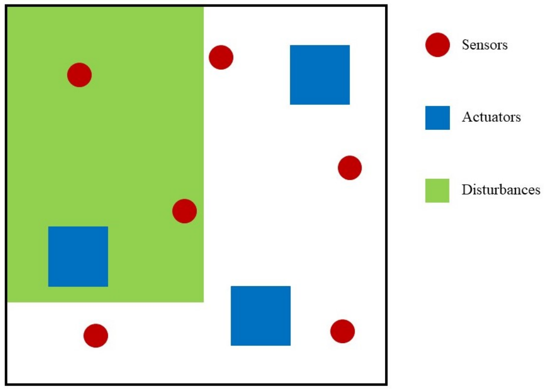
The ISS (input-to-state stability) concept [2] is investigated for a class of partial differential equations (parabolic systems). ISS means that the state norm is upper bounded by a continuous disturbance-dependent function, escaping to zero when the disturbance magnitude is nullified, whereas the effect of an arbitrary initial condition is captured by an additional term, depending on both the magnitude of the initial condition and on time, which asymptotically decays as time goes to infinity. The ISS for systems governed by partial differential equations has been previously investigated for specific configurations with boundary and/or point-wise sensing and actuation (see for example [3], [4]). In [1], ISS synthesis is developed for one-dimensional reaction-diffusion processes, located within a finite scalar spatial domain and governed by parabolic PDEs (heat equation) with a priori unknown spatially-varying heat capacity, diffusivity, and reaction coefficients. Moreover, in this reference, Y. Orlov, L. Perez, *et al.*, have developed a complete methodology for control law synthesis for specific 1D system with Neumann boundary conditions, with external disturbances, and with a finite number of non-collocated built-in-domain actuator-sensor.

The main objective of this PhD thesis is to extend the study to systems of dimension two in space (2D). The ISS synthesis to be developed is based on the Lyapunov approach, leading to the proportional output feedback. Particular attention will be paid to the optimal strategy for the actuators ensuring the stability (decay rate and disturbance attenuation level). Moreover, dual observer design must be investigated (based on Luenberger observer adapted to partial differential equations). The whole methodology will be developed for the reaction-diffusion process with uncertain spatially varying plant parameters. Effectiveness of the proposed synthesis will be analysed considering numerical simulations (finite element method will be implemented but will require numerous numerical adaptations).

A very original point of the requested work consists in the design of a real experimentation to show the attractiveness of the developed approach. It is indeed rare that conceptual developments are supported by experiments and results of real measurements. The thermal process (outline is briefly presented in the diagram below) will provide an attractive set up with several actuators (heating sources in blue) which are non-collocated with sensors (infrared thermometers in red). It is important to notice that spatial supports of both sensors and actuators are 2D. Moreover, disturbances are applied in a sub domain of the 2D geometry. Control law for actuators will be tested in several configurations to discuss method relevance and robustness of the control strategies.

There are many theoretical developments that can be addressed during this PhD thesis:

- general Robin boundary conditions of mixed type
- non linearities describing parameters which could be state dependent: heat capacity, diffusivity, reaction coefficients, heating fluxes (boundaries)
- specific cases of moving sensors and actuators
- control strategies adaptation required by actuators or sensors failures
- extension of the approach to other types of PDEs



Investigated geometry $(x,y)\in\Omega$ with disturbances and non-collocated sensors and actuators

$$\forall (x,y)\times t \in \Omega\times T$$

$$\rho(x,y)z_t(x,y,t) = \left(\theta(x,y)z_x(x,y,t)\right)_x + \left(\theta(x,y)z_y(x,y,t)\right)_y + \lambda(x,y)z(x,y,t) + f(x,y,t) + u(x,y,t)$$

$$\forall (x,y)\times t \in \partial\Omega\times T \qquad \frac{\partial z}{\partial \vec{n}} = 0$$

$$\forall (x,y)\in\Omega \qquad z(x,y,0) = z^0(x,y)$$

The schedule could be as follows:

- first semester:
  - bibliographic research about ISS (input-to-state stability) concept applied to parabolic partial differential equations
  - understanding of the thermal phenomena of conduction, convection, and radiation in order to master the modelling of heat transfers by a parabolic PDE associated with relevant boundary conditions.
- second semester:
  - theoretical developments for control law synthesis for specific 2D system with Neumann boundary conditions, with unknown spatially varying uncertain parameters and mismatched external disturbances, and with a finite number of non-collocated built-in-domain actuator-sensor.
  - numerical simulations based on finite element method
- third semester:
  - dual observer design and numerical tests
  - design of a real experimentation in a 2D geometry with non-collocated built-in-domain actuator-sensor (an experimental device based on infrared sensors and fixed heating disks should be developed)
- fourth semester:
  - the experimental bench: parametric identification (inverse heat conduction problem resolution) based on measurements and numerical simulations
  - validation of the ISS output feedback synthesis for PDEs in two dimensional geometries based on numerous experimental results.
- fifth semester and sixth semester:
  - exploratory investigation on new subjects (general Robin boundary conditions of mixed type, non-linear PDE systems, control strategies adaptation required by actuators or sensors failures, hyperbolic PDEs …)
  - writing and formatting of results

Main theoretical developments will be submitted in "*Automatica*" and "*IEEE Transactions on Automatic Control*". Experimental validation of the methodology on a real thermal process will be presented in "*Control engineering practice"* and *"Journal of process control".*
Studies could be presented in well-known conference: ECC, CDC, CPDE.

References:

[1] Y. Orlov, L. Perez, O. Gomez and L. Autrique, "*ISS output feedback synthesis of disturbed reaction-diffusion processes using non-collocated sampled-in-space sensing and actuation*", Automatica, 122, https://doi.org/10.1016/j.automatica.2020.109257, 2020.

[2] E. Sontag, "*Input to State Stability: Basic Concepts and Results*", In P. Nistri and G. Stefani (eds.) Nonlinear and Optimal Control Theory, Springer Verlag, Berlin, 163–220, 2008.

[3] I. Karafyllis and M. Krstic, "*ISS with respect to boundary disturbances for 1-D parabolic PDEs*", IEEE Trans. Aut. Contr., 61(2), 3712—3724, 2016.

[4] A. Pisano and Y. Orlov, "*On the ISS properties of a class of parabolic DPS' with discontinuous control using sampled-in-space sensing and actuation*", Automatica, 81, 447-454, 2017.

The PhD works will take place mainly at LARIS, in Angers (France), but travel to Mexico will be considered.

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