Internship in applied mathematics

Data assimilation using a 4d-var method.

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Location: Laboratoire Jacques-Louis Lions (LJLL), Sorbonne Université, 4 place Jussieu 75005 Paris.
Salary: around 575€ per month.
Starting date: February or March 2023.
Duration: 6 months.

Project: Data assimilation is an active research field in applied mathematics. It consists in combining some partial measurements from a system with the knowledge of its theoretical modeling in order to estimate the full system state. Applications exist in many industrial problems, for instance meteorology, non-invasive measurements or radars. Such a problem is particularly challenging when dealing with time-dependent partial differential equations.

Several methods have been developed to solve this problem. Among them, the 4d-var methods consist in rewriting the data assimilation problem in a variational formulation with time and space variables (the problem is then four dimensional in a 3d space setting). The system state is estimated as the trajectory solving an optimization problem for which the optimality criterion is computed according to the fit of the solution to the available data under the constraint of the partial differential equation modeling the problem under consideration.

A new class of methods has been developed recently to solve data assimilation problems [1]. These methods use so-called conditional stability estimates, see [2] for the heat equation. Compared to the literature, they ensure theoretical a priori error bounds, which strengthens the reliability of the results. Such a method has been developed by the supervisor to deal with the heat equation [3]. This version enables the use of high-order polynomials in space and in time. A C++ code has also been implemented.

The goal of this internship is to advance on the implementation aspects of the project. Two directions are considered:

• to develop a code to approach the state given by other equations (in particular the wave and the convection-diffusion equations),
• to run the code on a computing cluster to assess the performance of the method on more realistic test cases.

The successful candidate has some knowledge in the field of applied mathematics and informatics. In particular, he/she has some experience with at least one of those two programming languages: C++ and MATLAB. The topic of this internship could be considered to start a PhD thesis.