Sampling- and Homotopy-based Methods for Stochastic Optimal Control

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I. Context

From energy networks to space systems: complex Autonomous Systems (AS) have become pervasive in our society (e.g., Starek et al. 2015). In this context, the design of increasingly sophisticated methodologies for the control of AS is of utmost relevance, given that they regularly operate in uncertain circumstances and thus their dynamics must be modeled trough involved Itô-type stochastic differential equations of the form (e.g., Yong and Zhou 1999, Chapter 1.6)

$$\mathrm{d}x(t) = f(t, x(t), u(t))\,\mathrm{d}t + \sigma(t, x(t), u(t))\,\mathrm{d}W_t,$$

where x and u denote state and control variables respectively, mappings $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $\sigma : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{n\times d}$ define the drift and the stochastic diffusion respectively, and W_t is a d-dimensional Wiener process. In particular, to mitigate hazardous and possibly catastrophic uncertain perturbations during the decision-making process, one must optimally balance robustness with respect to the aforementioned perturbations with performance. These requirements boil down to efficiently and rapidly numerically solving Stochastic Optimal Control Problems (SOCP) of the form

$$\begin{aligned} &\min_{u\in\mathcal{U}} \quad \mathbb{E}\left[\int_0^T f^0(t, x(t), u(t))\,\mathrm{d}t\right] \\ &\mathrm{d}x(t) = f(t, x(t), u(t))\,\mathrm{d}t + \sigma(t, x(t), u(t))\,\mathrm{d}W_t \\ &x(0) = x_0 \in \mathbb{R}^n, \qquad \mathbb{E}\big[g(x(T))\big] \le 0, \end{aligned}$$

where $f^0 : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is a given cost function, $g : \mathbb{R}^n \to \mathbb{R}$ forces desired final conditions to hold true, and optimal controls are sought within some appropriate functional space $\mathcal{U}$.

II. State of the art

Several numerical paradigms have been proposed to solve SOCP when dynamics are linear and costs are convex, and relevant works include stochastic Riccati equations (Bismut 1976 and Tang 2003), primal-dual-based methods (Kuhn et al. 2009), and deterministic-equivalent reformulation (Bes and Sethi 1989). Unfortunately, the vast majority of modern AS often involves SOCP without any specific property, specifically, non-linear dynamics and non-convex costs are allowed. Although numerical approaches including dynamic programming (Yong-Zhou 1999, Chapter 4 and Pfeiffer 2020) have been studied to address the difficulties underneath such SOCP, their use remain limited to AS of small dimension. A methodology for leveraging techniques from the linear case to this general setting was very recently proposed in Bonalli et al. 2021. The development of computationally efficient numerical strategies remains challenging and is still intensively investigated in the community.

III. Proposed approach and goal of the internship

During this six-month internship, the candidate will develop and implement a new promising numerical scheme to solve general SOCP, which relies upon homotopy methods (e.g., Allgower-Georg 1990). The basic idea of homotopy methods is to solve a difficult problem by parameter deformation, step by step starting from a simpler problem. Thanks to their reliability, versatility, and theoretical guarantees, in the last decade they have seen a particular surge of interest in the control community (e.g., Trélat 2012). Homotopy methods are thus well-suited to design tractable schemes for SOCP, especially when combined with appropriate stochastic sampling techniques (Shapiro et al. 2014, Chapter 5).
The goal of the internship is twofold. First, through our attentive guidance, the candidate will devise an appropriate homotopy scheme where uncertainty, i.e, the stochastic diffusion σ, is introduced step by step starting from a deterministic problem. Then, she/he will numerically implement the aforementioned scheme in a MATLAB or Python formalism and run validating tests.

IV. Desired profile

The successful candidate has solid background in applied mathematics, in particular in stochastic processes and/or optimization and control. Good MATLAB or Python coding skill is a valuable perk.

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