Stage Master 2:
Mathematical models for thermal radiation in phase change problems

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Funded internship, to start in March/April, for 6 months (900euros/month)

Can be pursued with a PhD.

Description of the project:

The Resolving heat transfer is a crucial step in modelling many engineering applications. Heat is transmitted through three interactive mechanisms namely conduction, convection and radiation. In general, modelling heat transfer involves modelling these mechanisms as well as the interaction between them. In this proposal, we develop a class of highly efficient and accurate numerical models for radiation in applications that encounter a phase change in the material. From a mathematical point of view, a full radiative heat transfer model consists of integro-differential equations which depend on time, space, direction and frequency variables. These equations are therefore extremely difficult to solve, especially when coupled with the energy transport equations. This complexity makes the numerical simulation complicated and computationally expensive. Therefore, the development of mathematical models and numerical algorithms is critical and it would be novel and plausible in this research area. In our proposed work, we first develop a class of mathematical models for convection-radiation phase change problems using the simplified PN approximations for radiative heat transfer. The main advantage of this approach is that the conduction-radiation equations are transformed into a system of parabolic-elliptic equations independent of the angular directions. The coupled systems provide an accurate representation of radiation in conductive phase change problems not only for optically thick medium, but also when the regime is not largely diffusive, and even when the opacities are discontinuous. Next we extend the developed mathematical models for conduction-radiation phase change problems to include convection flows. Most existing works in convection-radiation problems neglect thermal radiation in convective medium mainly because it involves complicated mathematical models. However, in high temperature systems and when the buoyancy forces exist, the coupling convection-radiation cannot be neglected in the phase change problems and it greatly influences the thermal behaviors of the flow. Therefore, we propose an efficient conduction-convection-radiation phase change model that couples the conduction-radiation phase change model with the incompressible Navier-Stokes equations accounting for buoyancy effects using a Boussinesq approximation. Nonlinearity of the material is considered by allowing the flow, heat and radiative coefficients to depend on the temperature field.

Including convection to our modelling would increase the complexity of the whole conduction-convection-radiation phase change system. Therefore, design and implementation of new, fast and accurate numerical algorithms for solving the conductive-convective-radiative phase change problems are needed. In the current study, we develop a mixed finite element method for the space discretization and second-order implicit time stepping scheme for the time integration. The fully discretized system is reformulated into a nonlinear fixed point problem for which a Newton-type solver is developed. To increase the efficiency of the proposed method we also implement a dynamic mesh adaptation algorithm for solving this phenomenon using an error estimator accounting for all variables in the model. Anisotropic mesh adaptation is one of our main expertise and has not been implemented for conductive-convective-radiative phase change problems. The proposed algorithm would reduce substantially the number of degrees of freedom necessary to obtain accurate thermal predictions while capturing the main radiative features at low computational cost and memory consumption.

The performance of the proposed methodology will be assessed and analyzed on several test examples, including melting of gallium, cooling of class, scattering, isotropic and anisotropic media, strongly heterogeneous temperatures and absorption coefficients, etc. Our results will be obtained by first SPN1, but up to SPN3 model will be employed for the radiation. In some cases, a comparison between the obtained results and those obtained using the full radiative heat transfer equation will be performed.

Objectif de stage:

- Develop simplified mathematical models for radiative heat transfer in phase change.
- Establish numerical analysis for the developed partial differential equations with and without phase change.
- Implement the finite element method for solving the developed models.
- Perform numerical simulations and validations for some well-establish benchmarks.
- Apply the developed methods for solving problems in engineering.

Applicant requirements

The student should have knowledge on numerical analysis, PDE's analysis and also programming finite element methods in FreeFEM++ or Feel++.