Investigation of Size-spectrum Models in Marine Ecosystems

We propose an internship in the field of partial differential equations (PDEs) applied to biology, with emphasis on the modelling of the biological phenomenon. The aim is to analytically investigate a mathematical model for the size distribution of individuals in a marine ecosystem by asking questions as existence- and long-time convergence of solutions as well as limiting behaviour with respect to model relevant parameters.

Trophical interactions between animals in the ocean were matter of interest since the 70' with first size-measurements of individuals taken and investigated by Sheldon et. al in [4, 5] . Since then, size-based ecosystem modelling was discovered as a powerful tool to have a deeper insight into impacts of human- and environment driven changes on the marine ecosystem. Predation behaviour between single individuals in the huge ecosystem give rise to this large scale patterns in size distribution of animals observed by marine biologists [1], a perfect example of a phenomenon widely known as emergence.

Based on first modelling attempts to be found in [3], we propose to investigate a deterministic jump-growth model of Boltzmann type, aiming to capture this emergence phenomenon. The equation of interest is derived from individual based dynamics governed by a stochastic process. It is based on the assumption that binary interactions between individuals in the ecosystem take place, i.e. a predator feeding on a prey, which then results in growth of the predator with assimilating a certain (usually very small) amount of its prey's mass as well as plankton production. Difficulties occur in showing validity of the mesoscopic model within the whole size spectrum as well as overcoming its mathematical challenges, which occur in proving existence and analytical properties of solutions.

Within the internship the candidate should first get familiar with the just mentioned topic and model approach, which then leads to addressing questions of the model's analytical properties. Further, also numerical simulations of the model can be addressed, in order to support expected or proved analytical behaviour. Mathematical methods used will be borrowed form the realm of PDE-theory, functional analysis, including entropy methods and perturbation analysis. Also limiting behaviour with respect to model parameters, describing specific biological settings, possible model extensions as well as different model approaches can be addressed in a next step. This will contribute to the improvement of mathematical descriptions of marine life, the overall aim of the project.

The internship should be performed at:

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under the supervision of Laura Kanzler together with Benoît Perthame and Benoît Sarels. The duration is planned to be 4 to 5 months in the upcoming spring semester. A usual gratification financed by FORMAL (From ObseRving to Modelling oceAn Life) can be expected.

References

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[3] S. Datta, G. W. Delius, R. Law, *A Jump-Growth Model for Predator–Prey Dynamics: Derivation and Application to Marine Ecosystems*, Bulletin of Mathematical Biology (2010) 72(6):1361-82, DOI: 10.1007/s11538-009-9496-5

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[5] R. W. Sheldon, W. H. Sutcliffe Jr., and M. A. Paranjape, *Structure of Pelagic Food Chain and Relationship Between Plankton and Fish Production*, J. Fisheries Res. Board Can. 34, 2344-2353.