Nonlinear, higher-order asymptotic homogenization of an elastic composite bar with application to wave propagation

Master-level internship, 2023
Institut Jean le Rond d’Alembert, Sorbonne Université

Context and objectives

Effective properties of periodic composite materials can be efficiently predicted using homogenized models. Asymptotic homogenization, exploiting the multi-scale nature of problems of interest (typically, long-wavelength wave propagation) is a powerful tool to rigorously derive the coefficients of the effective models. In particular, higher-order expansions can capture specific effects, including wave dispersion, size-effects, symmetry properties (for 2D or 3D), localization etc. that do not appear in the models at leading order.

For linear waves in periodic media, asymptotic homogenization is a well-established tool, that produces second-order dispersive models and can be complemented with boundary or transmission conditions to address bounded domains and interfaces, see [Cornaggia and Guzina, 2020] for a 1D configuration. On the other hand, non-linear problems, involving e.g. finite strains elasticity and nonlinear constitutive laws [Andrianov et al., 2011] or nonlinear interfaces between linear materials [Bellis et al., 2021], are more technical and much less explored.

A recent, energy-based approach has been proposed in the context of dimension reduction of nonlinear structures and discrete homogenization [Lestringant and Audoly, 2020]. By working directly on the energy, this latter method allows for significant simplifications, in particular when deriving higher-order contributions. Besides, the method is variational and allows one to obtain the energy of the effective model directly. An application of this method to linear waves in 1D periodic media, as addressed in [Cornaggia and Guzina, 2020], is ongoing and shows promising results.

This master project will aim at extending this method to nonlinear elasticity, still focusing on a 1D periodic bar problem. As in [Andrianov et al., 2011], material nonlinearities will be first considered to validate the approach by retrieving the leading-order model, and high-order contributions will then be explored to describe the wave propagation more accurately, in the spirit of [Bellis et al., 2021], see Figure 1.

Main steps and methods

The main steps will involve:

— a bibliographic study of asymptotic approximations to model waves in periodic continua,
— the extension of the current linear formulation to account for nonlinear effects,
— the derivation of an effective model at leading order : effective leading order energy and formulation of the wave equation by calculus of variations,
— an analysis of this effective model : is it well-posed?
— a numerical validation that will involve implementing the initial (microstructured) and the effective IBVPs and comparing their predictions,
— repeating the previous steps (3-5) for higher-order energy contributions,
— writing an internship report and preparing a scientific presentation of the results.

Mainly theoretical and semi-analytical results are expected for the 1D energy, possibly computed using a symbolic calculation language such as wolfram mathematica. The validation step is likely to involve solving non-linear wave problems numerically. The chosen software / language and numerical methods are not fixed yet and may depend on the applicant taste and expertise.
Figure 1 – Waves in a 1D periodic array of non-linear interfaces: superposition of the velocity field \( V_h \) computed in the microstructure and its first-order approximation \( V^{(1)} \), for two different central frequencies of the initial signal, corresponding to two values of the asymptotic parameter \( \eta \). The macroscopic features of the wave-field are well-recovered, including the sharpening of the wavefront, but the dispersive effects (secondary oscillations) are not: a second-order model is needed to capture these. From [Bellis et al., 2021].

Practical information

**Time:** 5 to 6 months, start expected about March 2023.

**Location:** Institut d’Alembert, Sorbonne Université, campus Pierre et Marie Curie, Paris, France.

**Salary:** Sorbonne Université internship stipend (\( \approx \) 570€/ month).

**Expected profile:** The candidate should be a master student in applied maths, acoustics, mechanical engineering or related fields. Knowledge of PDE analysis, asymptotic methods or wave physics of composite materials would be much appreciated. He or she should have a strong taste for theoretical work and programming.

**Candidacy:** The candidates must send CV (including recommendations if possible), grade records and cover letter to both supervisors. An interview may be programmed. *All questions are welcome!*

**Supervisors:**
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References


