Internship proposal:
Aggregative nonconvex optimization

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Abstract

This proposal deals with a class of nonconvex optimization problems, which can be analyzed and adressed numerically with the help of a mean-field relaxation. The objective of the internship is to develop and implement novel dual-based methods for the resolution of these problems. The successful candidate will be funded by a PGMO project and will have the opportunity to interact with all its members.

Context The internship will focus on optimization problems of the following abstract form:

$$\inf_{x=(x_1,\ldots,x_N)\in\prod_{i=1}^N X_i} J(x) := f\Big(\frac{1}{N}\sum_{i=1}^N g_i(x_i)\Big) + \frac{1}{N}\sum_{i=1}^N h_i(x_i). \qquad (\mathcal{P})$$

An interpretation is a follows: a coordinator must assign decision variables to a (large) set of N agents. The term $h_i(x_i)$ represents a cost specific to agent i. The term $g_i(x_i)$ represents the contribution of agent i to a "common good" (referred to as "aggregate"), defined as the average of the contributions of all agents.

This structure naturally arises in energy management problems involving a large set of small production (or consumption) units, whose production profiles should be optimized over time. In this context, the aggregate is the total production of energy and the function f typically penalizes a variation of the total production with respect to a prescribed production profile. An example is given in [1], many others, arising from various application fields, can be found in [2].

Mathematical aspects Problem $(\mathcal{P})$, in its full generality, is difficult to analyse and to solve numerically: it is non-convex, non-differentiable, and large-scale.

The ongoing research, realized within the PGMO project, has allowed us to prove that problem $(\mathcal{P})$ can be well approximated by the following relaxed problem:

$$\inf_{\mu=(\mu_i)\in\prod_{i=1}^N \mathcal{P}(X_i)} \tilde{J}(\mu) := f\Big(\frac{1}{N}\sum_{i=1}^N \int_{X_i} g_i(x_i)\,\mathrm{d}\mu_i(x_i)\Big) + \frac{1}{N}\sum_{i=1}^N \int_{X_i} h_i(x_i)\mathrm{d}\mu_i(x_i), \qquad (\tilde{\mathcal{P}})$$

under appropriate smoothness assumptions on f. It can be attacked with the conditional gradient algorithm. We currently investigate a variant of that algorithm which generates at each iteration a candidate $x^{(k)}$ for the original problem, rather than a candidate $\mu^{(k)}$ for the relaxed problem.

Objectives The main goal is to propose, investigate, and develop numerical methods based on the dual problem to $(\tilde{\mathcal{P}})$. Some key features to be leveraged are the strong convexity of the associated dual criterion and its decomposability.

Besides the development of an efficient dual method, the challenges to be taken up in this internship concern the reconstruction of a relaxed primal solution and eventually an approximate solution to $(\mathcal{P})$.

Further developments of the work could also focus on variants of $(\mathcal{P})$ involving coupling constraints, their relaxation and their resolution.

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Desired profile The successful candidate is a second-year master student or a third-year student in an engineering school. He/she has a strong interest in the analysis and numerical resolution of optimization problems. Advanced knowledge in convex analysis and numerical methods will be an advantage.

Practical information The intern will be hosted at the L2S (Laboratoire des Signaux et des Systèmes, CentraleSupélec). He/she will be mainly supervized by Laurent Pfeiffer and will take part to the regular meetings with the members of the PGMO project funding the internship:

- Frédéric Bonnans (Senior researcher, Inria-Saclay and CentraleSupélec)
- Kang Liu (PhD student, Inria-Saclay and CentraleSupélec)
- Nadia Oudjane (Senior research engineer, EDF R&D)
- Laurent Pfeiffer (Researcher, Inria-Saclay and CentraleSupélec)
- Cheng Wan (Researcher, EDF R&D).

Applications (CV and cover letter) should be sent to laurent.pfeiffer@inria.fr. Duration: up to six months.

References

[1] Beaude, O., Benchimol, P., Gaubert, S., Jacquot, P. (2020). Oudjane, N. A privacy-preserving method to optimize distributed resource allocation. *SIAM Journal on Optimization*, 30(3), 2303-2336.

[2] Wang, M. (2017). Vanishing price of decentralization in large coordinative nonconvex optimization. *SIAM Journal on Optimization*, 27(3), 1977-2009.