Study of an acoustic/transport splitting scheme on staggered grid for homogeneous two-phase flows models

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We are interested in simulating two-phase flows into the core of a Pressurized Water Reactor. Such flows are generally at low Mach number. For the present internship, we will use a mixture model called the HRM model which is described in [AHH09]. Our goal is to develop a new numerical method to approximate its solutions. The system of equations can be written under the form of an hyperbolic system of equations of conservation laws with source terms, that is :

$$\partial_t U + \nabla_{\mathbf{x}} \cdot F(U) = S(U), \quad t > 0,\ \mathbf{x} \in \mathbb{R}^3,$$

where U is the vector of the conservative unknowns, F the convective fluxes and S the vector of source terms. The new method we want to develop is based on a finite volume approach and two main ideas.

The first one consists in separating the acoustic (fast) part and the transport (slow) part of the system thanks to a classical splitting technique. This strategy is inspired by the work presented in [CGK17]. The authors use a colocated finite volume method, where all the unknowns of the system are discretized at the center of the mesh cells, in order to solve the two sub-systems. An implicit time-based strategy for the acoustic part is also presented and allow to get a CFL restriction based only on the slow phenomenon.

The second idea of our new numerical method, and which will differ from [CGK17], is based on a staggered discretization of the unknowns : the vectorial unknown (the velocity of the fluid) is discretized at the edges (or faces in 3D) of the mesh and the scalar ones (the thermodynamical variables) are located at the center of the cells. Staggered approaches have been extensively used in the past decades for simulating incompressible flows but also for low compressible flows as it is the case in our two-phase flows applications. The latter was first described in [HA68] and is also known as the *ICE* method. More recent works including time-explicit formulation have also been done as in [HLN13].

The intern will have to first theoretically derive the numerical method and then code a 1D and/or 2D time-explicit version (using Python language for instance). Finally, if numerical tests show promising results, a time-implicit version of the new method could also be tested.

References

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