Prediction of confinement effects on the number of Covid-19 outbreak in Algeria

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Introduction

- On 25 February, Algeria laboratory-confirmed its first case of coronavirus 2 (SARS-CoV-2), an Italian man who arrived on 17 February.
- On 2 March, Algeria confirmed two new cases of SARS-CoV-2.

Province	First case	Confirmed cases	Deaths	Recoveries
Blida	1 March 2020	799	107	133
Algiers	13 March 2020	507	108	261
Oran	21 March 2020	243	13	50
Sétif	19 March 2020	197	16	2
Ain Defla	24 March 2020	183	5	0
Constantine	22 March 2020	178	9	0
Tipaza	22 March 2020	140	23	0
Béjaïa	17 March 2020	128	14	1
Tizi Ouzou	12 March 2020	104	14	37
Bordj Bou Arréridj	16 March 2020	98	19	0
Tlemcen	23 March 2020	94	6	0
Total		**4,006**	**450**	**1,779**
Latest update: 30 April 2020[117]				

FIGURE: COVID-19 cases in some Algerian cities.

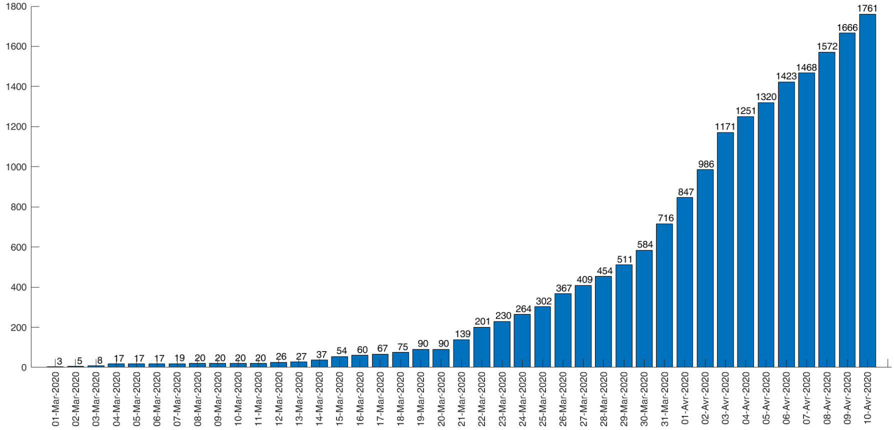


FIGURE: COVID-19 cases in Algeria.

$$I(t) = I_0 e^{\lambda t}$$

$$\lambda \approx 0.16$$

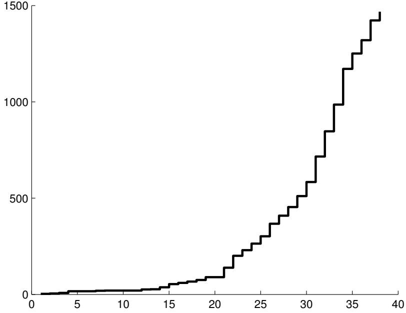


FIGURE: Cumulative cases of COVID-19 in Algeria

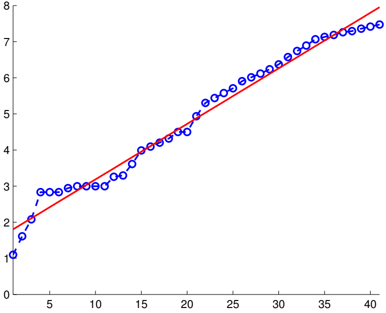


FIGURE: Neperian logarithm of the cumulative number of cases and linear regression lines.

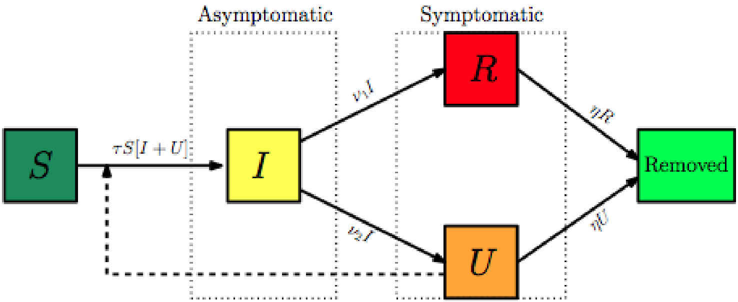


Figure 1. Diagram flux.

FIGURE: From : Z.Liu, P. Magal, O. Seydi, G. Webb. Understanding Unreported Cases in the COVID-19 Epidemic Outbreak in Wuhan, China, and the Importance of Major Public Health Interventions, Biology 2020.

$$\frac{dS}{dt} = -\beta S(t)I(t), \tag{1}$$
$$\frac{dE}{dt} = \beta S(t)I(t) - kE(t), \tag{2}$$
$$\frac{dI}{dt} = kE - \alpha I, \tag{3}$$
$$\frac{dR}{dt} = \alpha I, \tag{4}$$
$$S(t) + E(t) + I(t) + R(t) = 1, \tag{5}$$

$$\mathcal{R}_0 = \frac{\beta}{\alpha}.$$

we fix $1/k = 5$ [1] $and \quad \alpha = 1.$ [2]

1. T. Kuniya. Prediction of the Epidemic Peak of Coronavirus Disease in Japan, 2020, Journal of clinical medicine, 2020, 9, 789, doi :10.3390/ jcm9030789

2. N. Bacaër, Un modèle mathématique des débuts de l'épidémie de coronavirus en France. HAL Id : hal-02509142.

At the beginning of the outbreak, $S(t) \cong 1$

$$\frac{dE}{dt} \cong -kE + \beta I,$$
$$\frac{dI}{dt} \cong kE - \alpha I.$$

The predicted exponential growth rate, λ correspond to the largest eigenvalue of the Jacobian :

$$J = \begin{bmatrix} -k & \beta \\ k & -\alpha \end{bmatrix}.$$

$$\lambda = \frac{-(k+\alpha) + \sqrt{(k-\alpha)^2 + 4k\beta}}{2}.$$

Thus, it is possible to estimate β and $\mathcal{R}_0$ using the data set, we get,

$$\mathcal{R}_0 \approx 2.09 \ \ , \ \beta = 2.09.$$

$$\frac{dE}{dt} \cong -kE + \beta I, \tag{6}$$

$$\frac{dI}{dt} \cong kE - \alpha I. \tag{7}$$

Adding (6) and (7), we obtain

$$\frac{d(E+I)}{dt} = (\beta - \alpha)I = (\mathcal{R}_0 - 1)\frac{dR}{dt}.$$

Hence, at the beginning of the epidemic, we obtain

$$E(t) + I(t) + R(t) \approx \mathcal{R}_0 R(t). \tag{8}$$

The final sizes of S_∞, R_∞ when no control measures are put into place

Integration of the sum of the three equations (1), (2) and (3) from 0 to ∞ gives

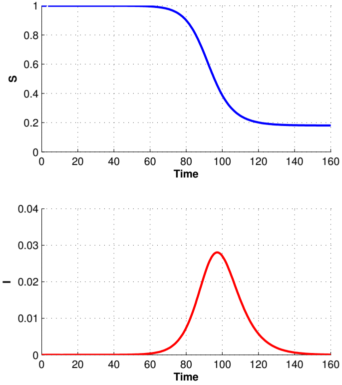
$$R_\infty = 1 - S_\infty = \alpha \int_0^\infty I(t)\,dt. \tag{9}$$

Division of the equation (1) by S followed by integration from 0 to ∞ gives

$$-\log \frac{S_\infty}{S_0} = \beta \int_0^\infty I(t)\,dt = \mathcal{R}_0 \Big(1 - S_\infty\Big). \tag{10}$$

Equation (10) gives implicitly S_∞ and therefore also the final epidemic size $R_\infty = 1 - S_\infty$.
At the beginning of the epidemic, the initial fraction of infected people I_0 is small compared with the population size, so $S_0 \approx 1$, formula (10) can be rewritten using $S_\infty = 1 - R_\infty$ as

$$1 - R_\infty \approx e^{-\mathcal{R}_0 R_\infty}. \tag{11}$$

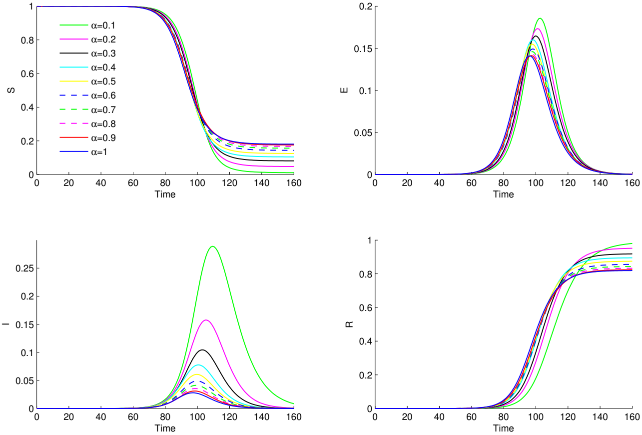


Solutions of $SEIR$ system (1)-(5), $S_0 = 1 - 10^{-7}$, $E_0 = 0$, $I_0 = 10^{-7}$, $R_0 = 0$. The number of new cases in Algeria will peak from around late may to early june and up to 82% of the Algerian population will likely contract the coronavirus if no measures were imposed

Sensitivity to paramater α

- At the start of the epidemic, we imagined that the proportion of asymptomatic patients was low, around 15 to 20%.
- Taking $\alpha = 1$ we assumed that almost all infected are symptomatic and that they are isolated very quickly on average after one day.
- More recently, observations have shown that the proportion of asymptomatic patients could be around 50% (Diamond Princess, Charles de Gaulle aircraft carrier).
- If 50% of the infected are asymptomatic with an average duration of infection of 10 days while the infected with symptoms are quickly isolated in 1 day, the average duration of stay in compartment I will be a little more than 5 days with a α of about 0.2.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
β	0.46	0.64	0.82	1,008	1.188	1.36	1.54	1.72	1.90	2.09
R_0	4.6	3.24	2.76	2.52	2.37	2.28	2.21	2.16	2.12	2.09



α=0.1
α=0.2
α=0.3
α=0.4
α=0.5
α=0.6
α=0.7
α=0.8
α=0.9
α=1
S
E
I
R
Time

Taking into account protective measures in the model

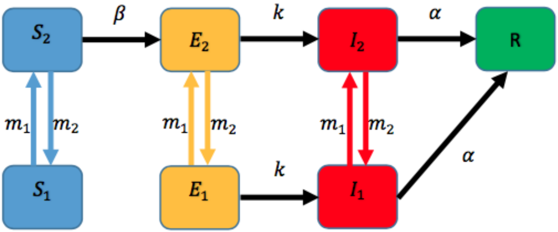


FIGURE: Flow diagram of the transmission dynamics of COVID-19.

The following complete model describes the time evolution of the epidemic :

$$\frac{dS_1}{d\tau} = m_2 S_2 - m_1 S_1, \tag{12}$$

$$\frac{dS_2}{d\tau} = m_1 S_1 - m_2 S_2 - \varepsilon\Big(\beta S_2(t) I_2(t)\Big), \tag{13}$$

$$\frac{dE_1}{d\tau} = m_2 E_2 - m_1 E_1 + \varepsilon\Big(- kE_1), \tag{14}$$

$$\frac{dE_2}{d\tau} = m_1 E_1 - m_2 E_2 + \varepsilon\Big(\beta S_2(t) I_2(t) - kE_2\Big), \tag{15}$$

$$\frac{dI_1}{d\tau} = m_2 I_2 - m_1 I_1 + \varepsilon\Big(kE_1 - \alpha I_1\Big), \tag{16}$$

$$\frac{dI_2}{dt} = m_1 I_1 - m_2 I_2 + \varepsilon\Big(kE_2 - \alpha I_2\Big), \tag{17}$$

$$\frac{dR}{d\tau} = \varepsilon\Big(\alpha I_1 + \alpha I_2\Big), \tag{18}$$

where τ is the fast time, $t = \varepsilon\tau$ is the slow time and where $\varepsilon << 1$ is a small dimensionless parameter.

$$
\begin{cases}
\frac{dS_1}{d\tau} = m_2 S_2 - m_1 S_1 \\
\frac{dS_2}{d\tau} = m_1 S_1 - m_2 S_2 + \varepsilon\Big(- \beta S_2(t) I_2(t)\Big) \\
\frac{dE_1}{d\tau} = m_2 E_2 - m_1 E_1 + \varepsilon\Big(- kE_1) \\
\frac{dE_2}{d\tau} = m_1 E_1 - m_2 E_2 + \varepsilon\Big(\beta S_2(t) I_2(t) - kE_2\Big) \\
\frac{dI_1}{d\tau} = m_2 I_2 - m_1 I_1 + \varepsilon\Big(kE_1 - \alpha I_1\Big) \\
\frac{dI_2}{d\tau} = m_1 I_1 - m_2 I_2 + \varepsilon\Big(kE_2 - \alpha I_2\Big) \\
\frac{dR}{d\tau} = \varepsilon\Big(\alpha I_1 + \alpha I_2\Big)
\end{cases}
$$

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\frac{dE_1}{d\tau} = m_2 E_2 - m_1 E_1 + \varepsilon \Big(- k E_1 \Big) \\
\frac{dE_2}{d\tau} = m_1 E_1 - m_2 E_2 + \varepsilon \Big(\beta S_2(t) I_2(t) - k E_2 \Big) \\
\frac{dI_1}{d\tau} = m_2 I_2 - m_1 I_1 + \varepsilon \Big(k E_1 - \alpha I_1 \Big) \\
\frac{dI_2}{d\tau} = m_1 I_1 - m_2 I_2 + \varepsilon \Big(k E_2 - \alpha I_2 \Big) \\
\frac{dR}{d\tau} = \varepsilon \Big(\alpha I_1 + \alpha I_2 \Big)
\end{cases}
$$

Aggregation of variables methods

$\varepsilon = 0$

$$S_1^* = vS \quad , \quad S_2^* = uS, \quad E_1^* = vE \quad , \quad E_2^* = uE, \quad I_1^* = vI \quad , \quad I_2^* = uI,$$

$$v = \frac{m_2}{m_1 + m_2} \qquad\qquad u = (1 - v)$$

$$\frac{dS}{dt} = -\beta_1 S(t)I(t), \tag{19}$$

$$\frac{dE}{dt} = \beta_1 S(t)I(t) - kE(t), \tag{20}$$

$$\frac{dI}{dt} = kE - \alpha I, \tag{21}$$

$$\frac{dR}{dt} = \alpha I, \tag{22}$$

where $\beta_1 = u^2\beta$.
Valid when the reduced system is structurally stable and for small value of $\varepsilon << 1$:

By integrating between T and ∞, we obtain the following result :

$$\log \frac{S(\infty)}{S(T)} = -u^2 \mathcal{R}_0 \Big(R(\infty) - R(T) \Big), \tag{23}$$

and this leads to the form

$$1 - R(\infty) = S(T) e^{-u^2 \mathcal{R}_0 \Big(R(\infty) - R(T) \Big)}. \tag{24}$$

Using the fact that $S(T) = 1 - (E(T) + I(T) + R(T)) = 1 - \mathcal{R}_0 R(T)$ we obtain

$$1 - R(\infty) \approx \Big(1 - \mathcal{R}_0 R(T) \Big) e^{-u^2 \mathcal{R}_0 \Big(R(\infty) - R(T) \Big)}, \tag{25}$$

which still cannot be solved explicitly.

Since $u^2\mathcal{R}_0 < 1$ and $0 < R(\infty) - R(T) << 1$, the approximation $e^{-x} \approx 1 - x$ gives

$$1 - R(\infty) \approx \Big(1 - \mathcal{R}_0 R(T)\Big)(1 - u^2\mathcal{R}_0\Big(R(\infty) - R(T)\Big). \tag{26}$$

We finally obtained

$$R(\infty) \approx \mathcal{R}_0 R(T)\frac{1 - u^2}{1 - u^2\mathcal{R}_0}. \tag{27}$$

When $u \to 0$, (corresponding to lockdown measures), the final size of the epidemic will be

$$R(\infty) \approx \mathcal{R}_0 R(T). \tag{28}$$

$$\mathcal{R}_0(t) = \begin{cases} \mathcal{R}_0 S(t) & \text{if} \quad t < T. \\ u^2 \mathcal{R}_0 S(t) & \text{if} \quad t \geq T. \end{cases}$$

We note

$$\mathcal{R}_0^\infty = \lim_{t\to\infty} \mathcal{R}_0(t) \approx u^2 \mathcal{R}_0 S(\infty) = u^2 \mathcal{R}_0 (1 - R(\infty)).$$

The effective reproduction number at ∞ is given as

$$\mathcal{R}_0^\infty \approx u^2 \mathcal{R}_0 \left(1 - \mathcal{R}_0 R(T) \frac{1-u^2}{1-u^2\mathcal{R}_0}\right). \tag{29}$$

If $\mathcal{R}_0^\infty < 1$, the epidemic is declining and is under control (vice versa if $\mathcal{R}_0^\infty > 1$). Consequently, there exists an unconfinement threshold u^* below which u must be chosen in order to stop the epidemic. Above that threshold, the epidemic is likely to reoccur :

$$u^* :\approx \frac{1}{\sqrt{\mathcal{R}_0}} \approx 0.69. \tag{30}$$

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R_0	4.6	3.24	2.76	2.52	2.37	2.28	2.21	2.16	2.12	2.09
u^*	0.46	0.55	0.60	0.63	0.64	0.66	0.67	0.68	0.68	0.69

Simulations (containment and protection threshold)

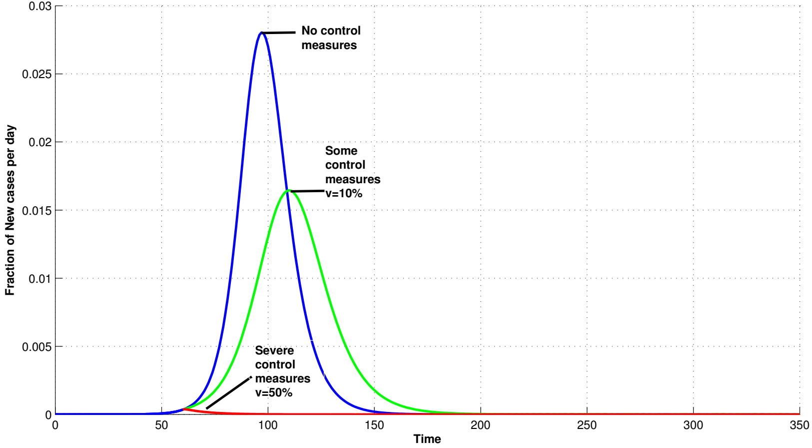

FIGURE: How Control Measures Could Slow the Outbreak. Without action (blue). Some control measures ($v = 10\%$, green), severe control measures ($v = 50\%$, red).

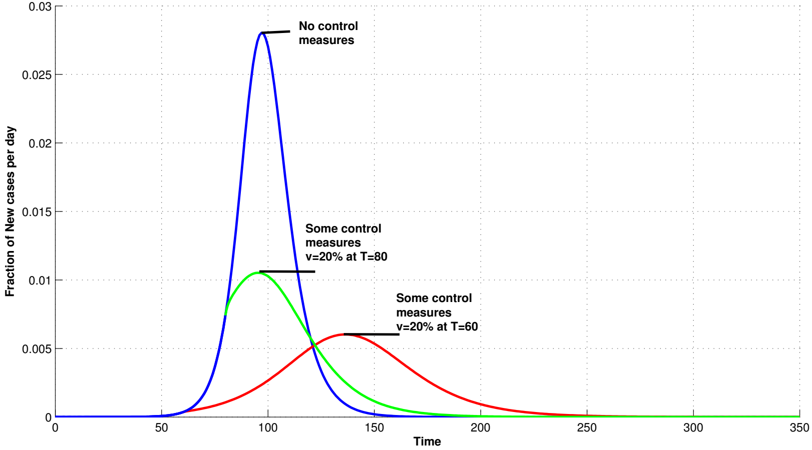

FIGURE: How Control Measures Could Slow the Outbreak. Without action (blue). Some control measures at $T = 60$ ($v = 20\%$, red), some control measures at $T = 80$ ($v = 20\%$, green)

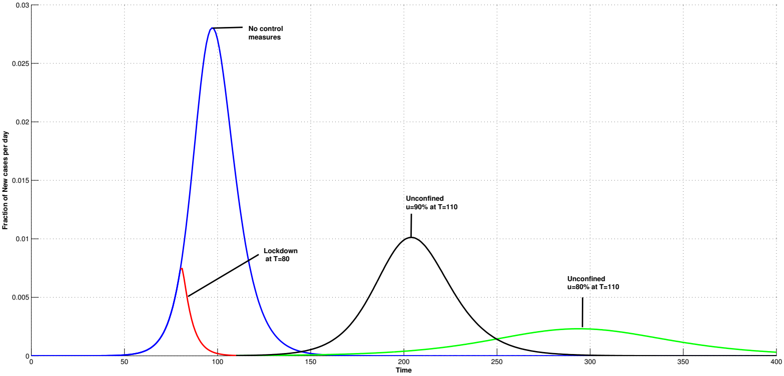

FIGURE: How Control Measures Could Slow the Outbreak. Lockdown ($v = 100\%$, red) at $T = 80$ followed with unsuccessful unconfinement, either $u = 0.8$ or $u = 0.9$ at $T = 110$.

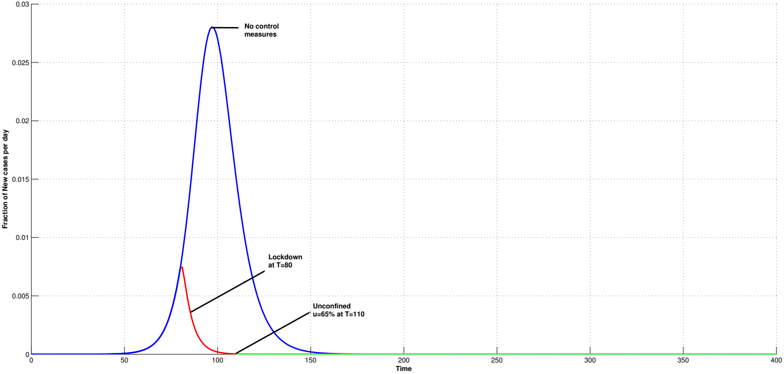

FIGURE: How Control Measures Could Slow the Outbreak.Lockdown ($v = 100\%$, red) at $T = 80$ followed with successful unconfinement at $T = 110$ ($u = 0.65$, green).

Perspectives

- Take into account different compartments corresponding to places (schools, work, shops, public transport, ...)
- Evaluate the effects of masks, social distancing... on the rate of contamination (parameter u)