A new model for image restoration

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in collaboration with

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http://www.realeyes3d.com

Paris, december 2009
Realey3D company overview

- General Presentation
- Our strategy: Camera Phone can do more than taking pictures
Founded 2001, 15 staff.

Offices in Paris (HQ), Hong Kong, San Francisco, Tokyo.

Privately held:
- 3M A-round 2003
- 7.5M B-round 2005

Publishing after patenting, Publishing when cannot patent

38 international patents
our goal

the world of mobile image acquisition

photography

- image improvement
- picture editing
- picture storage
- picture messaging
- etc.

We enable “visual intelligence at hand” and enhance the value of camera phone

- message personalization
- self content creation
- improved user experience
- improved productivity
- increased data traffic

- USER GENERATED CONTENT
- handwritten messaging
- document viewing
- mobile scanning
2 Products & Realeyes3D
   Technology, Products & services
Core Techs

**Ink Extraction**

- Mobile Scanning: thru ink extraction technology
- 24 patents (worldwide)
- **Service solution**: Qipit®
- **Embedded Products**:
  - Qipit White®
  - Visual Cortex®
  - Digitizer®
  - w-Postcard®
  - Clipper®,
  - MagicWanda®
Core Techs

Motion navigation

- thru camera apparent motion estimation
- 9 patents

**Products/Trademarks:**

- Motion Cortex®,
- MotionLens®
- 4 mini-games in the Nintendo DSi, browser integration ...

http://www.higp.jp/katamuku/index.html
Core Techs

Image restoration
- thru our unique deblurring technologies
- 3 patents
- application to barcode restoration and decoding
- Products/Trademarks: Xwalk
- application to document restoration (documents, V card ...)


A new model for image restoration
Image restoration
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A starting problem: barcode Image Restoration

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- **1D barcode** is widely used, especially for manufactured goods (billions of products) representation of data.
- **1D barcodes** are used for check-out and inventory control (UPC/EAN).
- **2D barcodes** represent data in patterns of squares, dots, etc. within images.
- Recent mobile phones have built-in camera, making virtually every cameraphone a barcode scanner. But shooting too far or too close?
  - too far result in a lack of resolution
  - too close result in a blurred image
Problem statement: is it possible to help cameraphones to read 1D barcodes? More precisely: can we decode a blurred barcode image shot by a cameraphone?
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We need some a priori knowledge about the degradation.
This is an inverse problem
We need a Mathematical description of how to blur an image:

\[ u_{\text{obs}} = u_{\text{orig}} \ast k + \eta \]

Goal: Given \( u_{\text{obs}} \), recover both \( u_{\text{orig}} \) and \( k \) up to a noise \( \eta \).
The mathematical model

- $\Omega \subset \mathbb{R}^2$: the domain of the image
- $u : \Omega \rightarrow \mathbb{R}$: the original image.
- $u_0 : \Omega \rightarrow \mathbb{R}$: the observed image (a degradation of $u$).

\[ u_0 = Ku + n, \]

- $n$: is the noise (a random variable),
- $K$ is the blur operator (linear). In practice $K = k \star$:
  - $k = \delta_0$ (dirac): no blur (denoising problem)
  - $k = k_0 e^{-\theta|x|^2}$: a Gaussian blur.
  - $k = k_0 1_{\{|x|<R\}}$: out-of-focus.
An approximation of $u$ is given by solving the minimization problem

$$
\min_u \int_\Omega |Ku - u_0|^2 dx + \lambda \int_\Omega \varphi(|\nabla u|) dx.
$$

Formally, $u$ satisfies the equation

$$
K^* Ku - \lambda \text{div} \left( \frac{\varphi'(|\nabla u|)}{|\nabla u|} \nabla u \right) = K^* u_0.
$$

which writes

$$
K^* Ku - \lambda \left( \frac{\varphi'(|\nabla u|)}{|\nabla u|} u_{TT} + \varphi''(|\nabla u|) u_{NN} \right) = K^* u_0.
$$

with $N = \frac{\nabla u}{|\nabla u|}$ and $T.N = 0$. 
Thus, the function $\varphi$ could be chosen such that

- Encourage **smoothing** (isotropic diffusion) in regions of **weak variations** of $u$ ($\nabla u \approx 0$).
- Direct the diffusion along the edges and not across them (to preserve edges).

$$
\lim_{t \to +\infty} \frac{t \varphi''(t)}{\varphi'(t)} = 0.
$$

**Example 1:** $\varphi(t) = \sqrt{1 + t^2}$

**Example 2:** $\varphi(t) = t$ (Total variation model).

(the usual function $\varphi(t) = t^2$ does not work: edges are not preserved).
There are two situations:

1. Restoring image $u$, with a known blurring $K$ (as the denoising problem)
2. Restoring both $u$ and $k$, with some noise

Let us start with the first problem: restoration with a known blurring $K$:

$$
\min_u E(u) = \int_{\Omega} |Ku - u_0|^2 dx + \lambda \int_{\Omega} \varphi(|\nabla u|) dx.
$$
The continuous problem

Usualy, we look for a solution $u$ living in the space

$$BV(\Omega) = \{ v \in L^1(\Omega) \mid \int_\Omega |Du| < \infty \},$$

where

$$\int_\Omega |Du| = \sup \left\{ \int_\Omega u \cdot \text{div} \phi \, dx ; \phi \in C^1_0(\Omega)^2, |\phi|_\infty \leq 1 \right\} ,$$

Theorem ($\varphi(t) = t$)

Suppose that $K : L^2(\Omega) \mapsto L^2(\Omega)$ is continuous and $K1 \neq 0$. Then, the problem

$$\min_{u \in BV(\Omega)} E(u) = \int_\Omega |Ku - u_0|^2 dx + \lambda \int_\Omega |Du| .$$

admits one and only one solution.

This theorem can be extended with minor modification to a larger class of functions $\varphi$.
Image: a collection of indexed pixels \((i, j), \ i \leq N, \ j \leq M\) and an intensity \(u \in X\) with \(X = \mathbb{R}^N \times \mathbb{R}^M\).

Define

- The inner product:
  \[
  \langle u, v \rangle = \sum_{i=1}^{N} \sum_{j=1}^{N} u_{i,j}v_{i,j}, \quad \|u\| = \langle u, u \rangle^{1/2}.
  \]

- The gradient \(\nabla u = (u_{i+1,j} - u_{i,j}, \ u_{i,j+1} - u_{i,j})_{i \leq N, j \leq M}\).

- The divergence: for \(p \in X^2\), \(\text{div} p \in X\) is such that
  \[
  \forall u \in X, \ \langle \text{div} p, u \rangle = -\langle p, \nabla u \rangle.
  \]

A few calculus gives

\[
\text{div} p \approx (p_{i,j}^{(1)} - p_{i-1,j}^{(1)} + p_{i,j}^{(2)} - p_{i,j-1}^{(1)}).
\]
The discrete energy writes

\[ E(u) = \frac{1}{2} \| Ku - u_0 \|^2 + \lambda J(u) \]

where

\[ J(u) = \sum_{i,j} \varphi(||(\nabla u)_{i,j}||). \]

In the sequel, we suppose that \( \varphi \) is strictly convex.
A descent algorithm (without line-search subalgorithm)

Let \( \phi(t) = \varphi(\sqrt{t}) \). For each \( u \in X \), set

\[
w(u) = K^*(Ku - u_0) - 2\lambda \text{div}(\phi'(|\nabla u|^2)\nabla u).
\]

- Initial data: \( n = 0, \ u^{(0)} = u_0 \).
- Step 1: if \( w(u^{(n)}) = 0 \), then stop.
- Step 2: choose \( d^{(n)} \) such that 
  \[
  |d^{(n)}| = 1 \quad \text{and} \quad \langle w(u^{(n)}), d^{(n)} \rangle \geq \tau |w(u^{(n)})| \quad (\tau > 0 \text{ is a constant}).
  \]
- Step 4: \( u^{(n+1)} = u^{(n)} - \alpha^{(n)} d^{(n)} \) where

\[
\alpha^{(n)} = \frac{\langle w(u^{(n)}), d^{(n)} \rangle}{\|Kd^{(n)}\|^2 + 2\lambda \sum_{i,j} \phi'(|(\nabla u^{(n)})_{i,j}|^2)(\nabla d^{(n)})_{i,j}|^2}.
\]

\( n \leftarrow n + 1 \). Go to Step 1.

**Proposition**

*If \( \phi \) is concave, then the sequence \((u^{(n)})\) converges to the minimizer of \( E(u) \) on \( X^2 \).*
Deblurring with an unknown blur

In practice, the blur function $k$ is often unknown. The problem we consider becomes

$$
\min_{k,u} E(k, u) = \int_{\Omega} |k \ast u - u_0|^2 dx + \lambda \int_{\Omega} \varphi(|\nabla u|) dx.
$$

To be tractable, $k$ is described by few parameters. For an out of focus blur, $k$ is of the form

$$
k(r) = \frac{1}{\pi r^2} 1_{B_r}(x),
$$

Here, $r$ denotes the radius of the out of focus. The problem writes

$$
\min_{r>0,u} E(r, u) = \int_{\Omega} |k \ast u - u_0|^2 dx + \lambda \int_{\Omega} \varphi(|\nabla u|) dx.
$$
Deblurring with an unknown blur

This problem is a non-convex problem. It can be solved hierarchically as following

- **Initial data:** \( u^{(0)} = u_0, \ r^{(0)} = r_0, \ n = 0. \)
- **Step 1:** find \( u^{(n+1)} \) minimizing \( E(r^{(n)}, u) \) (see previous algorithm)
- **Step 2:** find \( r^{(n+1)} \) minimizing \( E(r, u) \) (a 1D problem)
- **Step 4:** if \((r^{(n+1)}, u^{(n+1)})\) is solution then stop. Else \( n \leftarrow n + 1 \) and go Step 1.

<table>
<thead>
<tr>
<th>On a significant bench</th>
<th>Decoded</th>
<th>False positive</th>
<th>Not decoded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia phone : N70 ; original</td>
<td>0 %</td>
<td>0 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Nokia phone : N70 ; deblurred</td>
<td>85 %</td>
<td>4 %</td>
<td>11 %</td>
</tr>
</tbody>
</table>
A new model

If $k$ is an unknown out-of-focus blur. Then we have:

- $k$ is radial $k(x) = k(|x|)$,
- $\int_{\Omega} k(x)dx = 1$,
- $k$ has a compact support $\text{supp}k \subset [0, R]$.

Then, for each sufficiently smooth function $u$

$$k \ast u \approx \sum_{k=0}^{N} R^{2k} A_k u,$$

for $N$ sufficiently large. Here $A_k$ denotes a linear operator for each $k \in \{0, \ldots, N\}$.

Finally, we get the new model

$$
\min_{R,u} E(R, u) = \int_{\Omega} \left| \sum_{k=0}^{N} R^{2k} A_k u - u_0 \right|^2 dx + \lambda \int_{\Omega} \varphi(|\nabla u|) dx.
$$
A comparison: this new model and the classical blur model

Example: original barcode
A comparison: this new model and the classical blur model

Example: blurring operation: convolutive model vs our model; \( r = 8, N = 495 \)
A comparison: this model and the classical blur model; zoom

Example: blurring operation: convolutive model vs our model; \( r = 8, N = 495 \)
**Example of barcode restoration in high level of blur and noise:**

<table>
<thead>
<tr>
<th>Ideal</th>
<th><img src="image1.png" alt="Barcode Image" /></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Blurred + Noisy</strong></td>
<td><img src="image2.png" alt="Barcode Image" /></td>
</tr>
<tr>
<td><strong>Restored</strong></td>
<td><img src="image3.png" alt="Barcode Image" /></td>
</tr>
</tbody>
</table>
Example of V card restoration

Blurred Vcard shot by a cameraphone (Nokia N70)
Example

- If we restore by part the previous image, we get:
- Blurred image, OCR can’t see any letter or number
Example

- If we restore by part the previous image, we get:
- Blurred image, OCR can’t see any letter or number
If we restore by part the previous image, we get:
- Deblurred image, properly analyse by OCR
  (ABBYY and CardIris)
Conclusion and perspectives
Image restoration : blind deblurring

- By using the camera preview and the image streaming, we can improve our actual embedded blind deblurring.
- For barcode restoration, using a discrete optimization, work in progress with Laurent Dumas (LJLL lab). Toward a joint deblurring and decoding?
- Port the new model in its embedded version and try (numerically) the anistropic case.
Thank you for your attention!